Objective Phase-Space Quantum Information-Capacity Mechanics

An Information Ontology and Mathematical Framework for Quantum Mechanics

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Abstract

Quantum mechanics predicts experimental measurement statistics with high precision, yet the ontological status of the wave function and the dynamical mechanism of single-outcome state reduction remain unresolved. Prevailing foundational interpretations inherit two classical continuum idealizations: pointlike particle coordinates requiring unbounded spatial localization precision and instantaneous wave-function projections requiring infinite information-transfer bandwidth. Here we formulate Objective Phase-Space Quantum Information-Capacity Mechanics (QICM), a real-field information-geometric framework in which the complete objective state of a microscopic system on a configuration manifold M, relative to any other physical system, is uniquely determined by a probability density field $\rho(q,t)$ and a canonically conjugate compact flow potential $[\Phi(q,t)]$. Axiom I enforces finite spatial Fisher information $I_F[\rho] < \infty$ and defines $[\Phi]$ as the argument of a flat $U(1)$ bundle section modulo h obeying quantized vortex-loop holonomy and codimension-1 nodal half-period jumps $\pi\hbar$, axiomatically resolving Wallstrom's hydrodynamic objection. Axiom II defines the dimensionless constant $j_0 \equiv 1/(4\pi)$ from the Bloch-sphere quantum-Fisher solid angle 4π, parameterizes $s_{\min} = hj_0 = \hbar/2$ and $\hbar = 2hj_0$, and postulates the reference resolution scale $j = 1/(4\pi)$ nat. Axiom III pairs the Fisher localization kinetic energy $T_{\text{info}}[\rho] = [\hbar^2/(8m)]I_F[\rho]$ with the Nyquist–Planck channel capacity bound $C_{\max,\alpha} = \Delta E_{\text{int},\alpha}/h$ within coupled Itô equations unifying closed Hamiltonian flow and continuous diffusive measurement. Under the Madelung–Kähler map $\psi = \sqrt{\rho}e^{i\Phi/\hbar}$, the compatible Kähler triple $(\Omega, G_{\text{FR}}, J)$ reconstructs projective Hilbert-space kinematics, self-adjoint operator commutators, Robertson–Schrödinger uncertainty bounds, stationary bound-state spectra, Born-rule outcome probabilities via bounded-martingale convergence, and nonselective linear Lindblad dynamics (Theorems 1–7). The framework extends consistently to Lorentz-covariant free scalar and electromagnetic fields, frame-bundle half-integer spin and hydrogen fine structure, Compton-scale Dirac–Born–Infeld ultraviolet regularization, and relativistic ensemble no-signaling (Theorems 8–11). By replacing continuum infinities with finite localization energy and bounded channel bandwidth, QICM links psi-ontic real fields to continuous measurement and yields a parameter-free circuit-QED protocol testing the predicted $\sqrt{\bar{n}}$ capacity ceiling against linear photon-number scaling (Predictions I–II).

Keywords: quantum foundations; information geometry; continuous quantum measurement; wave-function collapse; finite channel capacity.

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Extended synopsis

Objective Phase-Space Quantum Information-Capacity Mechanics (QICM) develops a real-field information-geometric formulation of quantum kinematics and continuous measurement built on three explicit principles (Section 3):

1. **Axiom I (Objective uncertainty states and flat line-bundle topology):** A microscopic physical system has no classical pointlike "true coordinate value" at any time $t \in \mathbb{R}$ as an objective ontological commitment rather than mere epistemic ignorance; without making assumptions about material constituents or internal structure, its complete objective physical state on a configuration manifold M relative to any other physical system is uniquely determined by the real statistical field pair $(\rho(\cdot,t),[\Phi(\cdot,t)]) \in \mathcal{M}_{\mathrm{ontic}}$. The density ρ carries finite spatial Fisher information $I_F[\rho] < \infty$, while the compact flow potential $[\Phi]$ is the argument of a flat $U(1)$ bundle section modulo h obeying quantized loop holonomy [Eq. (3.1)] around codimension-2 vortices and a $\mathbb{Z}_2$ half-period jump $\pi\hbar$ across codimension-1 nodal hypersurfaces [Eq. (3.2)], axiomatically closing Wallstrom's hydrodynamic objection.
2. **Axiom II (Bloch-sphere geometric constant and reference resolution quantum):** From the total quantum-Fisher solid angle $A_{\mathrm{QFI}}(S^2) = 4\pi$ of the two-level projective sphere $\mathbb{CP}^1 \cong S^2$, QICM defines the dimensionless constant $j_0 \equiv 1/(4\pi)$, parameterizes the spin-1/2 projection scale $s_{\min} \equiv hj_0 = \hbar/2$ and reduced Planck constant $\hbar \equiv 2hj_0 = h/(2\pi)$ with the empirical action quantum h, and postulates $j = 1/(4\pi)$ nat as the reference mutual-information scale for an elementary resolution step.
3. **Axiom III (Finite-bandwidth action–information dynamics):** Localization kinetic energy is given by the Fisher functional $T_{\mathrm{info}}[\rho] = [\hbar^2/(8m)]I_F[\rho]$ (selected under Euclidean invariance, scale covariance, and pointwise subsystem composition in Proposition 3.1), while each elementary diffusive measurement channel α obeys the postulated Nyquist–Planck capacity bound $\gamma_{\mathrm{info},\alpha}(t) \le C_{\max,\alpha}(t) \equiv \Delta E_{\mathrm{int},\alpha}(t)/h$, with additive composition $\sum_\alpha C_{\max,\alpha}$ across independent channels. Coupled Itô equations govern closed Hamiltonian flow and record-conditioned diffusive state reduction.

Sections 4–6 establish the core mathematical and physical results: Kähler equivalence between the Sasaki–Fisher–Rao cotangent geometry $(\Omega, G_{\mathrm{FR}}, J)$ on the reduced horizontal tangent space $\mathcal{T}^{\mathrm{red}}_{(\rho,[\Phi])}\mathcal{M}_{\mathrm{ontic}}$ and complex Hilbert-space operator algebra under the Madelung–Kähler map $\psi = \sqrt{\rho}e^{i\Phi/\hbar}$ (Theorems 1–2); Robertson–Schrödinger uncertainty relations and the Gaussian-reference covariance differential entropy bound $S_{\mathrm{state}}[\Sigma] \ge 1 - \ln 2$ (Theorem 3); the characteristic channel-resolution time scale $\Delta t_{\min}(j) = \hbar/(2\Delta E_{\mathrm{int}})$ for extracting a reference unit $\Delta I_{\mathrm{ex}} = j$ (Theorem 4); closed-limit Schrödinger equivalence, Ermakov–Pinney Gaussian packet dynamics, and the semiclassical Hamilton–Jacobi/Liouville limit (Theorem 5); bound-state oscillator, orbital angular-momentum, and hydrogen Rydberg spectra (Theorem 6); and bounded-martingale measurement collapse on finite QND branches (and up to finite detector resolution $\Delta_{\mathrm{det}} > 0$ for continuous observables), establishing Born-weight consistency $\mathbb{P}(m_k) = P_k(0)$, the nonselective linear Lindblad master equation, and the discrete Shannon mutual-information identity (Theorem 7), which are applied in Section 6 to double-slit duality, continuous-variable and spin-singlet CHSH violations, and Zeno dynamics.

Section 7 develops four extensions: Lorentz-covariant Madelung–Klein–Gordon equations and positive-definite free scalar and transverse electromagnetic field functionals (Theorem 8); half-integer frame-bundle holonomy, the coherent-section identity $\langle \boldsymbol{S}^2 \rangle = s(s+1)\hbar^2$, local two-fermion radial Fisher repulsion under an explicit spin–statistics compatibility assumption, and a phase-space evaluation of $O(\alpha_{\mathrm{fs}}^2)$ hydrogen fine structure (Theorem 9 and Corollary 9.1); a phenomenologically

calibrated Compton-scale Dirac–Born–Infeld regularization ($\xi_c = 1/8$) with a mass-shell-consistent covariant action (Theorem 10); and the second-order Itô drift of Fisher information, ensemble no-signaling, and a monitored boundary-channel capacity bound (Theorem 11). Section 8 formulates a matched-parameter dispersive circuit-QED protocol testing the $\sqrt{\bar{n}}$ capacity ceiling of Axiom III against the linear $\bar{n}$ scaling of standard stochastic master equations (Prediction I) and isolates conditional phenomenological signatures of Assumptions H3–H4 (Prediction II). Appendix A summarizes the logical hierarchy of empirical inputs, axioms, calibrations, and theorems.

1 Introduction and physical ontology: finite information capacity as a paradigm for physical reality

1.1 Infinite precision and infinite bandwidth in prevailing interpretations

Since Schrödinger formulated wave mechanics and Heisenberg and Born established matrix mechanics and the probabilistic interpretation in 1926, quantum mechanics has predicted experimental measurement statistics across every accessible scale. Yet competing interpretations continue to disagree over the ontological status of the wave function $\psi(q,t)$ and the dynamical mechanism of measurement collapse. Despite divergent philosophical commitments, prevailing interpretations share two idealized continuum limits inherited from classical mechanics:

1. **Unbounded spatial localization precision.** Classical point mechanics and de Broglie–Bohm hidden-variable theories [1–4] assign a microscopic particle a geometrical point coordinate $q_{\text{true}}(t) \in \mathbb{R}^3$, treating the exact specification of an infinite-precision real position at every instant t as physically realized (a continuum idealization also critiqued from an intuitionistic finite-information perspective by Gisin and Del Santo [5, 6]). In the Everett many-worlds interpretation (MWI) [7–10], the universal wave function Ψ evolves unitarily and supports an emergent decoherence branching structure [8]. Whereas a continuum of mathematical field values does not by itself entail unbounded accessible classical information [11], QICM adopts the physical premise that compressing a spatial probability density to vanishing uncertainty $\delta q \to 0$ relative to a reference scale L_0 requires both a logarithmically divergent relative specification $\ln(L_0/\delta q) \to \infty$ and—as the primary coordinate-invariant physical barrier in Axiom III—a divergent Fisher localization kinetic energy $T_{\text{info}}[\rho] = [\hbar^2/(8m)]I_F[\rho] \propto (\delta q)^{-2} \to \infty$, anchoring phase-space resolution to the symplectic cell h^d.
2. **Infinite channel bandwidth.** The von Neumann projection postulate represents conditioning on an ideal measurement outcome through an instantaneous state update [12, 13]—a zero-duration limit, $\tau_{\text{collapse}} \to 0$, that serves as a mathematical idealization rather than a physical model of apparatus coupling across all Copenhagen or pragmatist accounts [14–16]:

$$|\psi\rangle \longmapsto \frac{\hat{P}_k|\psi\rangle}{\|\hat{P}_k|\psi\rangle\|}.$$

Physically, any measurement extracts and records information through a coupling channel with a finite pure entanglement interaction energy spread ΔE_{int}. Acquiring a nonzero amount of information in zero time would demand an unbounded transfer rate, $C = dI_{\text{ex}}/dt \to \infty$, which QICM excludes via an independent finite-bandwidth postulate. While Shannon's noisy-channel coding theorem [17] and Holevo's accessible-information bound for quantum ensembles [11] constrain static or asymptotic channel capacities without directly fixing a Hamiltonian rate limit, QICM's dynamical capacity ceiling connects continuous quantum measurement theory [18–21] to the finite-resource rate and thermodynamic erasure bounds explored by Landauer, Bremermann, and Lloyd [22–24].

Remark (Physical intuition and interpretation: finite information capacity). Recognizing that any finite phase-space volume and finite interaction energy support only finite information storage capacity and finite transmission bandwidth unifies four foundational features of quantum mechanics within a single physical picture:

1. *Intrinsic uncertainty and zero-point energy.* Compressing $\rho(q)$ steepens its logarithmic edge gradients and increases $I_F[\rho]$, requiring a localization kinetic energy proportional to $I_F[\rho]$ [Eq. (3.6)]. Microscopic systems consequently resist point localization, retaining objective spatial extent and irreducible zero-point motion.

2. *Complex Hilbert space and noncommuting operators.* An extended objective field is characterized by both a density ρ, specifying where probability weight resides, and a flow potential $[\Phi]$, governing its transport. Simultaneous preservation of the canonical symplectic structure Ω and the statistical distinguishability metric G_{FR} endows real phase space with a Kähler complex structure, from which complex wave functions and operator commutators emerge naturally (Theorems 1 and 2).

3. *Discrete stationary spectra.* Because the compact circle-bundle flow potential acquires integer-h increments around closed loops and undergoes half-period jumps $\pi\hbar$ across standing-wave nodal surfaces, a bound probability fluid supports discrete topological eigenmodes analogous to the integral harmonics of a vibrating string (Theorem 6).

4. *Unique outcomes and finite collapse times.* Measurement acts as continuous Bayesian information locking constrained by the maximum bandwidth $C_{\max} = \Delta E_{\mathrm{int}}/h$. Along an individual experimental run, unbiased bounded-martingale updating drives the observable variance to zero, selecting a single eigenoutcome with Born probability $P_k(0)$ over a completion time bounded below by the channel capacity (Theorem 7 and Prediction I in Section 8).

1.2 Three central ontological commitments of QICM

Objective Phase-Space Quantum Information-Capacity Mechanics (QICM) is built on three physical commitments:

1. **Objective statistical fields replace geometrical points and a primitive complex wave function.** A microscopic physical system possesses no classical, geometrically pointlike "true coordinate value" at any time $t \in \mathbb{R}$; this absence is an objective ontological property rather than epistemic ignorance of underlying point coordinates. Setting aside assumptions regarding internal material substructure, QICM characterizes how a microscopic system manifests relative to any other physical system on its configuration manifold M: its complete objective state is uniquely determined by the real statistical field pair $(\rho(\cdot,t),[\Phi(\cdot,t)]) \in \mathcal{M}_{\mathrm{ontic}}$, comprising the normalized objective probability density $\rho(q,t) \geq 0$ and its canonically conjugate compact information-flow potential $[\Phi(q,t)]$. Its first moment defines the position centroid $\bar{q}(t)$, while the square root of the trace of its second central moment defines the objective spatial uncertainty $\Delta_q(t) > 0$, which likewise reflects an intrinsic ontic property dictated by finite phase-space Fisher information capacity rather than epistemic ignorance of an underlying point position. Within the Harrigan–Spekkens classification [25, 26], this real-field state characterization is psi-ontic by construction (Theorem 1), assigning distinct ontic field pairs to distinct pure states (whereas the Pusey–Barrett–Rudolph theorem [27] reaches psi-onticity under an additional preparation-independence postulate). Complementing Wheeler's "it from bit" program and operational reconstructions of quantum theory [28–32]—which likewise retain continuous reversible transformations, as in Hardy's fifth axiom [30]—QICM anchors state characterization directly in continuous real statistical fields on configuration space, with the

complex wave function $\psi(q,t) \equiv \sqrt{\rho(q,t)}e^{i\Phi(q,t)/\hbar}$ serving as their Kähler complex coordinate representation (Theorem 1).

2. **A Bloch-sphere geometric constant and a postulated reference information-resolution scale.** In the standard Braunstein–Caves normalization $ds^2_{\mathrm{QFI}} = 4ds^2_{\mathrm{FS}}$, the two-level projective manifold $\mathbb{C}P^1 \cong S^2$ has total solid-angle area 4π. Motivated by this projective geometry, QICM defines the dimensionless geometric constant $j_0 \equiv 1/(4\pi)$ and postulates the reference information-resolution quantum $j = 1/(4\pi)$ nat as the characteristic mutual-information scale of an elementary resolution step. Pairing j_0 with the empirical elementary action quantum h parameterizes the smallest nonzero intrinsic spin projection magnitude (the spin-1/2 scale) $s_{\min} \equiv hj_0 = \hbar/2$ and the reduced Planck constant $\hbar \equiv 2hj_0 = h/(2\pi)$ (whereas the eigenvalue spacing between adjacent projections within any spin multiplet remains $2s_{\min} = \hbar$).

3. **Measurement collapse is channel-driven diffusive state reduction with Nyquist–Planck bandwidth.** Unlike spontaneous-collapse models (GRW/CSL) that posit universal background noise in isolated systems, QICM is an interaction-driven measurement framework: a strictly isolated system ($\Delta E_{\mathrm{int}} = 0$) evolves unitarily with zero spontaneous heating, whereas coupling to an open measurement channel with pure entanglement energy spread $\Delta E_{\mathrm{int},\alpha} > 0$ induces diffusive state reduction governed by the monitored observable $M_\alpha(q)$ and bounded by the elementary channel capacity $C_{\max,\alpha} = \Delta E_{\mathrm{int},\alpha}/h$ in nat/s. Within the domain where solutions remain regular (established globally for finite QND branches and up to any finite resolution scale $\Delta_{\mathrm{det}} > 0$ for continuous observables in Theorem 7), a single system of coupled Itô equations encompasses both closed Hamiltonian flow ($\Delta E_{\mathrm{int}} = 0$) and open record-conditioned measurement collapse ($\Delta E_{\mathrm{int}} > 0$).

1.3 Scope of the framework

Sections 4–7 develop the mathematical and physical structure of QICM across five levels:

- **Kähler geometry and operator algebra (Theorems 1–2).** Compatibility between the canonical symplectic structure Ω and the rescaled Fisher–Rao metric G_{FR} induces the complex Hilbert-space Hermitian inner product, self-adjoint observable representations, the Lie-algebra isomorphism between real-field Poisson brackets and Heisenberg commutators, and unitary premeasurement symplectomorphisms for nonconfiguration observables.
- **Nonrelativistic mechanics and continuous measurement (Theorems 3–7).** In the nonrelativistic regime, the axioms yield multidimensional Robertson–Schrödinger uncertainty relations, the positive covariance-entropy bound in $\mathbb{R}^d$, the energy–time resolution limit, exact equivalence with the closed Schrödinger equation, and Ermakov–Pinney Gaussian packet breathing. Variational and topological quantization recover the one-dimensional harmonic-oscillator spectrum, three-dimensional orbital angular momentum $l(l+1)\hbar^2$, and the hydrogen Rydberg spectrum $E_n = E_1/n^2$ with n^2 degeneracy, while open-channel stochastic updating establishes variance decay, Born-rule outcome probabilities, and the nonselective linear Lindblad master equation.
- **Relativistic covariance and free quantum fields (Theorem 8).** Extending the framework to four-dimensional Minkowski spacetime reconstructs covariant hydrodynamic equations for positive-frequency scalar fields and positive-definite Schrödinger probability functionals for free scalar and Coulomb-gauge electromagnetic fields on the field configuration space $\mathcal{C} = \{\varphi(\boldsymbol{x})\}$, identifying Fock number excitations with Hermite–Gaussian nodal topology regularized by Gaussian cylindrical measures.

- **Intrinsic spin, identical-particle statistics, and fine structure (Theorem 9 and Corollary 9.1).** On the frame bundle $\mathbb{R}^3 \times SO(3)$, nontrivial $\mathbb{Z}_2$ holonomy yields half-integer spin sectors $s = k + 1/2$ with highest-weight projection $S_{\parallel} = s\hbar$, 2π spinor sign reversal, and the total spin-squared identity $\langle \boldsymbol{S}^2 \rangle = s(s+1)\hbar^2$ from Hopf-bundle Kähler–Fisher curvature. On the identical-particle quotient manifold $M_N = (\mathbb{R}^{3N} \setminus \Delta_{\mathrm{diag}})/S_N$, topological exchange holonomy (linked to spin via the compatibility condition in Theorem 9) generates Bose–Fermi sectors and repulsive Pauli Fisher information pressure, while Foldy–Wouthuysen evaluation reproduces the relativistic hydrogen fine-structure splitting.
- **Compton-scale regularization and many-body information dynamics (Theorems 10–11).** A phenomenological Dirac–Born–Infeld (DBI) ultraviolet regularization at the reduced Compton scale $\lambda\!\!\!^{-}_C = \hbar/(mc)$ enforces subluminal drift saturation at vortex cores and calibrates first-order nodal osmotic-energy saturation to the electron–positron rest-energy scale $2mc^2$. For composite and many-body systems, a second-order Itô drift identity for Fisher information establishes ensemble no-superluminal-signaling consistency alongside a monitored boundary-channel capacity bound on mutual-information extraction.

1.4 Interpretations, the Heptalemma, and entropic dynamics

The Heptalemma no-go theorem and diagnostic of DeBrota and List (2026) [33] provide a way to locate QICM among contemporary foundational approaches. The Heptalemma resolves classical realism into seven propositions: (1) *measurement realism*, according to which outcomes are facts with well-defined probabilities; (2) *non-relationalism*, according to which facts hold absolutely rather than only relative to an observer or reference system; (3) *non-fragmentation*, according to which all facts form one jointly consistent set; (4) *one world*, according to which a unique objective world exhausts reality; (5) *locality*, excluding instantaneous measurement effects at spacelike separation; (6) *measurement independence*, requiring statistical independence of settings and initial states; and (7) *non-solipsism*, admitting multiple observers. Their conjunction is incompatible with quantum predictions, including EPR–Bell–CHSH violations and the GHZ, Hardy, Leggett, and extended Wigner-friend no-go results [34–44], whereas any six are compatible. A consistent interpretation, including superdeterministic, retrocausal transactional, or indivisible stochastic alternatives [45–50], must identify which proposition it relinquishes.

Table 1 situates major interpretations within the diagnostic taxonomy of DeBrota and List [33], which classifies Copenhagen, MWI, relational quantum mechanics (RQM), consistent histories, and QBism according to whether they relinquish measurement realism, non-relationalism, non-fragmentation, or the single-world premise (recognizing that individual proponents and neo-Copenhagen commentators often contest these classifications). In terms of the seven atomic propositions of the Heptalemma, QICM retains (1) measurement realism, (2) non-relationalism, (3) non-fragmentation, (4) one world, (6) measurement independence, and (7) non-solipsism, and *relinquishes proposition (5) (Bell locality)*. More precisely, under Jarrett's decomposition of Bell locality into parameter independence plus outcome independence [51], QICM relinquishes *outcome independence* (since conditioning on a measurement record at A nonlocally updates the joint configuration-space field across an entangled ridge) while strictly satisfying *parameter independence* (relativistic ensemble no-signaling, Theorem 11) [33, 51]. Likewise, in extended Wigner-friend scenarios [37–39], QICM draws an objective physical distinction between a closed unitary premeasurement gate $\hat{U}_{\mathrm{pre}}$ ($\Delta E_{\mathrm{int}} = 0$ to external channels, which merely entangles the friend's probe with the system without collapsing the state) and an open diffusive measurement channel ($\Delta E_{\mathrm{int}} > 0$, which irreversibly locks a single global branch on $\mathcal{M}_{\mathrm{ontic}}$ and precludes superobserver interference).

Table 1: Interpretational comparison using the Heptalemma. Each interpretation is classified by the realist proposition it relinquishes and contrasted with QICM.

Interpretation	Ontological status of the quantum state	Relinquished Heptalemma proposition [33]	Collapse mechanism	Physical distinction of QICM
Copenhagen/ pragmatism [12–16, 52]	Predictive state assignments; Healey regards quantum probabilities as objective normative guidance	Measurement realism or non-fragmentation in the taxonomy of Ref. [33] (though pragmatist accounts retain objective normative probabilities)	Idealized projection, $\tau_c = 0$, in textbook rules; in Healey's account, reassignment is relative to accessible records rather than a physical jump	Hilbert space arises from Kähler geometry; the cut and instantaneous postulate are removed, giving testable capacity bounds on collapse times and exploratory discrete-resolution thresholds (Section 8).
MWI/Everett [7–10, 53]	Universal Ψ as sole objective reality, a "single macroworld" tree	Measurement realism in the diagnostic of Ref. [33]: all branches occur; decision-theoretic and self-location accounts nevertheless propose Born probabilities [9, 10]	No collapse; all branches persist with continuous real weights	Bounded martingale convergence in Theorem 7 gives $P_k(\infty) \in \{0, 1\}$ and objective Born probabilities, restoring measurement realism; j resolves branches.
RQM/Rovelli [37, 54, 55]	States are binary relations between systems, with no absolute quantum state	Non-relationalism: facts hold only relative to an interacting pair $\langle A, B\rangle$	Interactions establish relative facts; no absolute collapse criterion	Non-relationalism is retained: relative to any other physical system, a microscopic system manifests in the form characterized by the same objective pair $(\rho, [\Phi]) \in \mathcal{M}_{\text{ontic}}$, and collapse driven by ΔE_{int} is an objective physical process.
QBism [56–59]	An agent's subjective credence in future personal experience	One world (pluriverse QBism), or non-fragmentation (fragmentalist QBism)	Subjective Bayesian updating after new personal experience	One world and non-fragmentation are retained: S_{state} and I_F are physical quantities entering energy–momentum conservation.
Consistent histories/ contextuality [60]	Consistent history probability frameworks; sheaf and Kochen–Specker approaches analyze contextuality [61, 62]	Non-fragmentation in the diagnostic of Ref. [33]: incompatible frameworks cannot be combined into a single probabilistic description	Histories assign probabilities when consistency conditions hold, without requiring collapse; contextuality results specify no collapse dynamics	Theorem 2 attributes noncommutativity to premeasurement symplectic dynamics $\hat{U}_{\text{pre}}$; the ontic state remains one globally consistent Kähler field pair.

Interpretation	Quantum-state ontology	Relinquished proposition	Collapse mechanism	Physical distinction of QICM
de Broglie–Bohm [1–4, 63]	True point trajectories $q(t)$ and a nonlocal guiding wave	Locality: velocities depend instantly on global coordinates; nonequilibrium $\rho \neq \|\psi\|^2$ permits superluminal signaling	Empty packets cease to guide the point particle (effective collapse)	Infinite-precision trajectories are removed; $\rho(q,t)$ itself is ontic, excluding the nonequilibrium signaling loophole; the quantum potential is a Fisher-energy variation.
GRW/CSL spontaneous collapse [64–67]	Wave function with postulated stochastic localization: discrete GRW jumps or continuous CSL noise	Locality: nonlocal state reduction in standard formulations; standard nondissipative models also predict spontaneous heating	GRW uses random localization hits; CSL uses continuous stochastic evolution [64, 66]	No background white-noise parameters; isolated stationary states with $\Delta E_{\text{int}} = 0$ have zero spontaneous heating; the detector supplies all backaction energy (Theorem 11).
Subjective entropic dynamics/ Jaynes–Caticha [68, 69]	Epistemic probability describing incomplete knowledge	Epistemic antirealism, without a closed treatment of Bell nonlocality and continuous-measurement bandwidth limits	Closed Schrödinger dynamics is derived; projection updating remains	Density and capacity are objective; $C_{\max}$ and Compton ultraviolet regularization give a collapse-time lower bound.
QICM [27, 33, 45, 51]	Objective statistical field pair $(\rho(\cdot,t),[\Phi(\cdot,t)]) \in \mathcal{M}_{\text{ontic}}$ uniquely characterizing the state on M relative to any other system	Relinquishes proposition (5) (Bell locality) via outcome dependence, while retaining (1)–(4), (6)–(7) and parameter independence (ensemble no-signaling)	Channel-driven bounded martingale collapse with $C_{\max,\alpha} = \Delta E_{\text{int},\alpha}/h$ (Theorem 7)	Finite localization-energy regularity and postulated channel bandwidth connect unitary flow to diffusive state reduction without superluminal signaling (Theorem 11).

1.5 Relations to quantum hydrodynamics, Kähler geometry, and continuous measurement

The closed equations of QICM coincide with the hydrodynamic equations of Madelung (1927) [70], Takabayasi (1952) [2], and Nelson [71], and with the Fisher-information variational formulations of Reginatto (1998, 1999) [72, 73] and Hall and Reginatto (2002) [74]. Its cotangent Kähler geometry builds directly upon the geometric quantum mechanics of Kibble (1979) [75], Ashtekar and Schilling (1999) [76], Brody and Hughston (2001) [77], and the Sasaki–Fisher–Rao Madelung-transform geometry of Khesin, Misiołek, and Modin (2018, 2019) [78]. Under the complex coordinate map, its open-system equations for a monitored configuration observable correspond in the Markovian limit to the diffusive stochastic master equations (SMEs) of Belavkin, Gisin, Diósi, and Korotkov [18–21, 65, 79–82]. In the closed and unconstrained Markovian regimes, QICM is therefore mathematically equivalent to standard quantum mechanics plus continuous quantum filtering; its distinct foundational and physical content resides in four specific structural elements:

1. **Flat-bundle holonomy and first-order nodal jumps provide an axiomatic closure of Wallstrom's objection [83].** A single-valued real Madelung velocity potential neither explains quantized circulation nh nor excludes kink-like spurious solutions at radial or polar nodal hypersurfaces of oscillators and atoms. These codimension-one surfaces disconnect nodeless regions without admitting encircling loops. Complementing alternative regularity

and geometric proposals by Schmelzer [84] and Reddiger [85], QICM defines the potential as the argument of a flat $U(1)$ section modulo h and explicitly postulates both loop holonomy, Eq. (3.1), and the codimension-1 nodal hypersurface $\mathbb{Z}_2$ half-period jump, Eq. (3.2), thereby ensuring hydrodynamic–Schrödinger equivalence in arbitrary spatial dimensions.

2. **Hilbert-space kinematics and operator algebra are reconstructed from projective Kähler compatibility.** Whereas textbook mechanics introduces complex Hilbert space and noncommuting self-adjoint operators as primitive algebraic postulates, Theorems 1–2 construct the compatible Kähler triple $(\Omega, G_{\mathrm{FR}}, J)$ on the reduced horizontal tangent space of real Ontic Phase Space (quotienting out global phase shifts). The complex wave function, Hermitian inner product, and Lie-algebra isomorphism $[\hat{A}, \hat{B}] \leftrightarrow i\hbar\{A, B\}_\Omega$ emerge as the linearized coordinate representation in a globally single-valued complex chart.

3. **A postulated gauge-invariant Nyquist–Planck bandwidth bounds measurement information rates.** In a standard Belavkin stochastic Schrödinger equation, the coupling rate is an externally specified parameter whose formal divergence recovers instantaneous projection. After subtracting local mean-field shifts, QICM postulates that the pure entanglement interaction energy spread ΔE_{int} bounds the instantaneous mutual-information extraction rate via $\gamma_{\mathrm{info}}(t) \leq C_{\max} = \Delta E_{\mathrm{int}}/h$, connecting continuous measurement theory to energy-constrained information-gain and quantum speed limits [86–90]. This removes the infinite-bandwidth collapse idealization and yields testable bounds on continuous-variable compression times and discrete-readout horizons (Prediction I), alongside exploratory non-Markovian discrete-resolution effects (Prediction II).

4. **A phenomenological Compton-scale ultraviolet regularization tames vortex and nodal gradient singularities.** In standard nonrelativistic Madelung hydrodynamics, the irrotational drift speed $v = |\nabla\Phi|/m$ and local osmotic momentum $p_{\mathrm{osm}} = -(\hbar/2)\nabla \ln \rho$ diverge at vortex cores and first-order nodal surfaces. The DBI-type bounded-bandwidth effective functional of Theorem 10 provides subluminal vortex-core drift saturation at $\widetilde{\lambda}_C = \hbar/(mc)$ and calibrates the local osmotic-energy ceiling to the electron–positron rest-energy scale $2mc^2$.

To orient readers trained in standard Hilbert-space quantum mechanics, Table 2 provides a direct conceptual dictionary translating textbook wave-function and operator constructs into their real-field information-geometric counterparts in QICM.

Table 2: Conceptual bridge between textbook Hilbert-space quantum mechanics and Objective Phase-Space Quantum Information-Capacity Mechanics (QICM).

Textbook Quantum Concept	**QICM Real-Field Counterpart**	**Physical Mechanism and Intuition**
Complex wave function $\psi(q,t)$	Real field pair $(\rho(q,t), [\Phi(q,t)]) \in \mathcal{M}_{\mathrm{ontic}}$ via $\psi = \sqrt{\rho}e^{i\Phi/\hbar}$	ρ specifies objective spatial probability density; $[\Phi]$ (mod h) governs coherent probability transport (Axiom I, Theorem 1).
Imaginary unit i and inner product $\langle\psi_1|\psi_2\rangle$	Quarter-turn Kähler rotation J ($J^2 = -I$) and $G_{\mathrm{FR}} + i\Omega$	Preserving both Fisher distinguishability G_{FR} and symplectic action area Ω locks real tangent variations into a complex Hilbert space (Theorem 1).
Kinetic energy $\langle\hat{p}^2\rangle/(2m)$	Flow kinetic energy T_{flow} plus Fisher localization energy $T_{\mathrm{info}} = [\hbar^2/(8m)]I_F[\rho]$	Sharpening $\rho(q)$ steepens logarithmic edge gradients $\nabla \ln \rho$, creating osmotic pressure that resists point collapse (Axiom III, Theorem 3).

Textbook Quantum Concept	**QICM Real-Field Counterpart**	**Physical Mechanism and Intuition**
Canonical commutator $[\hat{q},\hat{p}] = i\hbar$	Real-field symplectic Poisson bracket $\{Q,P\}_\Omega = 1$	Shifting density ρ and tilting phase gradient $\nabla\Phi$ do not commute, enclosing a symplectic phase-space area $\hbar$ (Theorem 2).
Stationary eigenstate $\|n\rangle$	Standing-wave topological mode with n nodal jumps $\Delta\Phi = \pi\hbar$ or vortex windings $n_l h$	Single-valuedness on the compact circle fiber modulo h admits only discrete standing-wave harmonics (Axiom I, Theorem 6).
Wave-function collapse and Born's rule $P_k = \|\langle k\|\psi\rangle\|^2$	Unbiased Itô Bayesian updating with capacity $C_{\max} = \Delta E_{\text{int}}/h$	Zero-drift martingale competition drives variance $\Delta_M^2(t) \to 0$ over a finite duration while conserving $\mathbb{E}[P_k(t)] = P_k(0)$ (Axiom III, Theorem 7).

Remark (Ontology, the Heptalemma, and nonlocality without signaling). By characterizing the complete objective physical state of a microscopic system relative to any other physical system uniquely through $(\rho(\cdot,t),[\Phi(\cdot,t)]) \in \mathcal{M}_{\text{ontic}}$—without positing classical pointlike "true coordinate values" or making assumptions about material constituents or internal structure—Axiom I retains measurement realism, non-relationalism, non-fragmentation, a single world, measurement independence, and non-solipsism. Adopting the locality-relinquishing branch of DeBrota and List's Heptalemma [33] then requires explaining nonlocal quantum correlations without permitting superluminal signaling—a reconciliation achieved in QICM through three linked mechanisms:

1. **Jarrett decomposition [33, 51].** While the Fisher localization energy $T_{\text{info}}[\rho_{AB}]$ is additive on factorized product states $\rho_{AB} = \rho_A\rho_B$ [Proposition 3.1, Eq. (3.12)], an entangled density forms a nonfactorizable ridge in joint configuration space. Single-trajectory Bayesian updating across this shared ridge violates outcome independence, $P(A,B|a,b,\lambda) \neq P(A|a,\lambda)P(B|b,\lambda)$, reproducing the Tsirelson CHSH value $2\sqrt{2}$ (Section 6.2). Because a local observable at A satisfies $\nabla_{q_B} M_A(q_A) \equiv 0$, Theorem 11 [Eq. (7.10)] ensures $d\mathbb{E}[I_{F,B}[\rho_{AB}]]_{\text{meas}} \equiv 0$: monitoring A injects neither partial Fisher information nor backaction energy into spacelike-separated B, providing the dynamical foundation for parameter independence.

2. **Zero-drift martingale conservation and pointwise nonlinear cancellation.** In Eqs. (5.31) and (5.33), conditional state updating is driven by the zero-mean Wiener innovation $dW_t^{(A)}$ encoding the local readout record at A. For a distant observer at B without subluminal access to that classical record, the nonselective evolution under a deterministic bare coupling rate $\kappa_{\text{meas}}^{(A)}(t)$ [with $\gamma_{\text{info}} = 2\kappa_{\text{meas}}\Delta_M^2$, Eq. (3.10)] exhibits exact pointwise cancellation of all nonlinear state-mean terms $\bar{M}_t$ in the Itô differential $d(\psi\psi^*)$. Consequently, the reduced coherence kernel at B obeys an autonomous linear Lindblad equation independent of A's measurement settings [Eq. (7.11)], avoiding Gisin–Polchinski nonlinear signaling [91–93] (while any predictive extension beyond standard Born statistics remains subject to the free-choice assumptions of the Colbeck–Renner theorem [94]).

3. **Exclusion of the subquantum nonequilibrium loophole.** In point-particle pilot-wave theories, departures from quantum equilibrium ($\rho_{\text{particle}} \neq |\psi|^2$) allow superluminal signaling [63]. Because QICM posits no hidden point coordinates distinct from the objective density field $\rho(q,t)$ itself, this nonequilibrium signaling channel does not arise: equal-norm prior weights are fixed by envariance symmetry (Section 3.5), while coarse-grained sampling discrepancies relax via the relative-entropy H-theorem [Eq. (3.14)] whenever unresolved mixing satisfies its advection–diffusion and entropy-coercivity conditions.

2 Basic quantities, geometric–information constants, and levels of state representation

2.1 Physical definitions of the information quantities and constants

Before stating the axioms, we define the core information-theoretic and geometric quantities and explicitly distinguish the three distinct senses in which "information regularity and capacity" enter the framework: (a) state-level Fisher gradient regularity $I_F[\rho] < \infty$, (b) second-moment phase-space symplectic volume $\det \Sigma \geq (\hbar/2)^{2d}$, and (c) open-channel mutual-information rate $C_{\max,\alpha} = \Delta E_{\mathrm{int},\alpha}/h$.

1. **Geometric definition of the dimensionless constant j_0 and the postulated reference resolution scale j.** On the elementary two-level pure-state projective manifold $\mathbb{C}P^1 \cong S^2$, the standard Braunstein–Caves quantum Fisher metric $ds^2_{\mathrm{QFI}} = 4ds^2_{\mathrm{FS}} = d\theta^2 + \sin^2\theta\, d\phi^2$ contracts monotonically under completely positive trace-preserving (CPTP) maps [95–97] and has total solid-angle area $A_{\mathrm{QFI}}(S^2) = 4\pi$ (a convention-dependent normalization fixed by the factor of 4 relating ds^2_{QFI} to the Fubini–Study metric ds^2_{FS}). We define the dimensionless solid-angle geometric constant as the reciprocal of this area:

$$j_0 \equiv \frac{1}{A_{\mathrm{QFI}}(S^2)} = \frac{1}{4\pi} \approx 0.07957747. \tag{2.1}$$

For any fixed pure state ψ and any binary orthogonal projector $\hat{P}$ (such as testing whether the system lies inside or outside a measurable region Ω) for which both $\hat{P}\psi$ and $(\hat{I} - \hat{P})\psi$ are nonzero, the normalized two-dimensional span $\mathrm{span}\{\hat{P}\psi, (\hat{I} - \hat{P})\psi\}$ defines a projective submanifold isomorphic to $\mathbb{C}P^1 \cong S^2$, on which $j_0 = 1/(4\pi)$ is the geometric fraction of one steradian relative to the full sphere. Pairing j_0 with the empirical elementary action quantum h provides a geometric parameterization (rather than an a priori derivation) of the smallest nonzero spin projection magnitude $s_{\min} \equiv hj_0 = h/(4\pi) = \hbar/2$ and the reduced Planck constant $\hbar \equiv 2hj_0 = h/(2\pi)$:

$$j_0 \equiv \frac{1}{4\pi}, \qquad j = j_0\,\mathrm{nat} = \frac{1}{4\pi}\,\mathrm{nat}, \qquad s_{\min} \equiv hj_0 = \frac{\hbar}{2}, \qquad \hbar \equiv 2hj_0 = \frac{h}{2\pi}.$$

Throughout, information is measured with natural logarithms in units of nat. In Axiom II, we postulate $j = 1/(4\pi)\,\mathrm{nat}$ as the characteristic reference mutual-information unit for an elementary state-resolution step (changing to bits rescales both I_{ex} and j by $1/\ln 2$, $j \approx 0.1148\,\mathrm{bit}$, leaving $N_{\mathrm{info}} = I_{\mathrm{ex}}/j$ invariant).

2. **Spatial Fisher information: gradient regularity and localization kinetic energy.** For the objective probability density field on configuration space,

$$I_F[\rho] \equiv \int_{U_t} \rho |\nabla \ln \rho|^2\, d^d q = 4 \int_M |\nabla \sqrt{\rho}|^2\, d^d q < \infty.$$

Physically, $I_F[\rho]$ quantifies the edge sharpness of $\rho(q,t)$, and multiplying by $(hj_0)^2/(2m) = \hbar^2/(8m)$ gives the Fisher localization kinetic energy $T_{\mathrm{info}}[\rho]$. We emphasize that the condition $I_F[\rho] < \infty$ in Axiom I is a Sobolev H^1 regularity condition ensuring finite localization energy for each admitted state (for example, a narrow Gaussian of width $\sigma > 0$ has $I_F = \sigma^{-2} < \infty$ for every $\sigma > 0$). A state-independent upper bound on $I_F[\rho]$ arises only when a uniform energy ceiling $T_{\mathrm{info}}[\rho] \leq E_{\max}$ is imposed ($I_F[\rho] \leq 8mE_{\max}/\hbar^2$, which bounds spatial resolution by $\Delta_q \geq \hbar/\sqrt{8mE_{\max}}$) or when the Compton-scale DBI ultraviolet cutoff of Theorem 10 saturates the local gradient density.

3. **Gaussian-reference covariance differential entropy vs. exchanged channel mutual information.** The second-moment covariance entropy $S^{(d)}_{\mathrm{state}}[\Sigma] \equiv \frac{1}{2}\ln[((2\pi e)^{2d}\det\Sigma)/h^{2d}]$ is defined as the differential Shannon entropy of the reference Gaussian distribution sharing the system's $2d\times 2d$ phase-space covariance matrix Σ with respect to the dimensionless phase-space measure $d^dq\,d^dp/h^d$. It is a logarithmic measure of second-moment phase-space volume relative to the Planck cell h^d (carrying the convention-dependent Gaussian offset $(2\pi e)^d$) and must not be conflated with the von Neumann entropy $S_{\mathrm{vN}} = -\operatorname{Tr}(\hat\rho\ln\hat\rho)$ or thermodynamic Gibbs entropy: any pure one-dimensional Gaussian has $S_{\mathrm{vN}} = 0$ while saturating $\sqrt{\det\Sigma} = \hbar/2$ and $S^{(1)}_{\mathrm{state}}[\Sigma] = 1-\ln 2 \approx 0.3069\,\mathrm{nat}$. In contrast, $I_{\mathrm{ex}}(t) = \int_0^t \gamma_{\mathrm{info}}(\tau)\,d\tau$ is the accumulated classical mutual-information flux extracted by an external measurement channel.

4. **Two levels of phase-space geometry.** At the infinite-dimensional field level, the real pair $(\rho,[\Phi])$ defines the cotangent quotient

$$\mathcal{M}_{\mathrm{ontic}} = T^*\mathcal{P}_{\mathrm{phys}}(M)/h\mathbb{Z},$$

equipped with Ω and G_{FR} and called *Ontic Phase Space*. At the finite-dimensional moment level, the position–momentum covariance matrix Σ and its symplectic invariants define covariance phase space. Neither representation relies on a Wigner–Weyl quasiprobability distribution; local Wigner negativity in non-Gaussian states is fully compatible with the strict nonnegativity of the configuration-space density $\rho(q,t)\geq 0$, as the two quantities represent distinct mathematical transforms.

2.2 Global notation, unique definitions, and physical dimensions

Table 3: Global mathematical notation and dimensions. L, T, M denote length, time, and mass dimensions.

Symbol	Physical quantity	Definition, function space, dimensions, and interpretation
q, t, m, c	Configuration, time, mass, speed of light	$q = (q_1,\dots,q_d)^T \in \mathbb{R}^d$ has dimension L; $t\in\mathbb{R}$ has dimension T; $m>0$ has dimension M; the vacuum speed of light c has dimension LT^{-1}.
$h, j_0, j, \hbar, s_{\min}$	Planck constant, geometric constant, reference information unit, and action scales	$h>0$ is the empirical elementary action quantum, dimension ML^2T^{-1}; $j_0 \equiv 1/(4\pi)$ is dimensionless; $j = 1/(4\pi)\,\mathrm{nat}$ is the postulated reference information-resolution unit; $\hbar \equiv 2hj_0 = h/(2\pi)$; $s_{\min} \equiv hj_0 = \hbar/2$ is the smallest nonzero intrinsic spin projection magnitude (spin-1/2 scale).
$s, n_s, S_\parallel$	Spin quantum number, fiber weight, and highest-weight spin projection	$s = n_s/2$ and n_s are dimensionless; $S_\parallel = s\hbar = n_s s_{\min}$ has dimension ML^2T^{-1}. In the nontrivial $\mathbb{Z}_2$ spin sector, n_s is odd.
$\lambda_C, \widetilde{\lambda}_C$	Compton and reduced Compton wavelengths	$\lambda_C \equiv h/(mc)$ is the elementary Compton resolution scale; $\widetilde{\lambda}_C \equiv 2j_0\lambda_C = \hbar/(mc)$ is the reduced Compton ultraviolet cutoff, dimension L.
$\rho(q,t), \varrho(q,q';t), \varrho(x)$	Nonrelativistic probability density, two-point coherence kernel, and relativistic scalar modulus	$\sqrt{\rho(\cdot,t)} \in H^1(\mathbb{R}^d)$ and $\int_{\mathbb{R}^d}\rho(q,t)\,d^dq = 1$, dimension L^{-d}; $\varrho(q,q';t)$ is the two-point configuration coherence kernel with diagonal $\varrho(q,q;t) = \rho(q,t)$; in Minkowski spacetime $\mathbb{R}^{1,3}$, $\varrho(x) = \lvert\varphi(x)\rvert^2 \geq 0$ denotes the Lorentz scalar wave modulus.

Symbol	Physical quantity	Definition, function space, dimensions, and interpretation
$[\Phi(q,t)], \Phi(q,t)$	Compact information-flow potential and local representative	$[\Phi(q,t)]$ denotes the equivalence class modulo h ($\Phi \sim \Phi + nh$, $n \in \mathbb{Z}$) of the argument of a flat $U(1)$ bundle section on $U_t = \{q \mid \rho(q,t) > 0\}$, dimension ML^2T^{-1}; in local differential equations on U_t, $\Phi(q,t)$ denotes a smooth local representative branch.
$(\Omega, G_{\mathrm{FR}}, J)$	Ontic Kähler triple	Defined on the reduced horizontal tangent space $\mathcal{T}^{\mathrm{red}}_{(\rho,[\Phi])}\mathcal{M}_{\mathrm{ontic}} = \{(\delta\rho, \delta\Phi) : \int_M \delta\rho\, d^dq = 0, \int_M \rho\,\delta\Phi\, d^dq = 0\}$: $\Omega = \int_M \delta\rho \wedge \delta\Phi\, d^dq$; $G_{\mathrm{FR}} = \int_M [(\hbar/2\rho)\delta\rho_1\delta\rho_2 + (2\rho/\hbar)\delta\Phi_1\delta\Phi_2] d^dq$; $J(\delta\rho, \delta\Phi) = (-(2\rho/\hbar)\delta\Phi, (\hbar/2\rho)\delta\rho)$; $J^2 = -I$, $\Omega(u,v) = G_{\mathrm{FR}}(Ju, v)$.
$v(q,t), p_{\mathrm{osm}}(q,t)$	Local drift velocity and osmotic momentum	$v(q,t) \equiv \nabla\Phi(q,t)/m$ describes coherent flow, dimension LT^{-1}. $p_{\mathrm{osm}}(q,t) \equiv -\hbar j_0 \nabla \ln \rho(q,t) = -(\hbar/2)\nabla \ln \rho(q,t)$ describes the intrinsic gradient resisting compression, dimension MLT^{-1}.
$\bar{q}(t), \bar{p}(t), \Delta_q(t)$	Position mean, momentum mean, objective position uncertainty	$\bar{q}(t) \equiv \int_{\mathbb{R}^d} q\rho(q,t)\, d^dq$; $\bar{p}(t) \equiv \int_{\mathbb{R}^d} \nabla\Phi(q,t)\rho(q,t)\, d^dq$; $\Delta_q(t) \equiv [\int_{\mathbb{R}^d} \lvert q - \bar{q}(t)\rvert^2 \rho(q,t)\, d^dq]^{1/2}$ measures objective spatial extent.
$I_F[\rho]$	Spatial Fisher information	$I_F[\rho] \equiv 4\int_{\mathbb{R}^d} \lvert\nabla\sqrt{\rho(q,t)}\rvert^2 d^dq = \int_{U_t} (\lvert\nabla\rho\rvert^2/\rho) d^dq = \int_{U_t} \rho \lvert\nabla \ln\rho\rvert^2 d^dq$, dimension L^{-2}; measures edge sharpness and phase-space resolution capacity.
$\Delta_p(t), \sigma_{qp}(t)$	Momentum uncertainty and position–momentum covariance	$\Delta_p^2(t) \equiv 2m(T_{\mathrm{flow}} + T_{\mathrm{info}}) - \lvert\bar{p}(t)\rvert^2$; in one dimension $\sigma_{qp}(t) \equiv \int_{-\infty}^{\infty} \rho(q-\bar{q})(\partial_q\Phi - \bar{p}) dq$, measuring packet expansion or contraction correlations.
$\Sigma(t), D_\Sigma, S^{(d)}_{\mathrm{state}}$	Covariance matrix, symplectic area, state entropy	In one dimension, $D_\Sigma(t) \equiv \sqrt{\det\Sigma(t)} = \sqrt{\Delta_q^2\Delta_p^2 - \sigma_{qp}^2}$; in d dimensions, $S^{(d)}_{\mathrm{state}}[\Sigma] \equiv \frac{1}{2}\ln[((2\pi e)^{2d}\det\Sigma)/h^{2d}]$, in nat.
$\kappa_{\mathrm{meas}}, \gamma_{\mathrm{info}}, C_{\max}$	Bare measurement rate, extraction rate, capacity	$\kappa_{\mathrm{meas}}(t) \geq 0$ is a deterministic apparatus-controlled bare rate; $\gamma_{\mathrm{info}}(t) = 2\kappa_{\mathrm{meas}}(t)\Delta_M^2(t) = dI_{\mathrm{ex}}/dt \leq C_{\max} \equiv \Delta E_{\mathrm{int}}/h$ in nat/s; $\Gamma_{\mathrm{Nyq}} \equiv C_{\max}/j = 2\Delta E_{\mathrm{int}}/\hbar$ is the maximal quantum-step rate.
$\hat{H}^{\mathrm{ent}}_{\mathrm{int}}, \Delta E_{\mathrm{int}}$	Pure entanglement interaction operator and pure entanglement interaction energy standard deviation (gauge-invariant definition)	Removing local marginal Hamiltonian partial traces gives $\hat{H}^{\mathrm{ent}}_{\mathrm{int}}(t) \equiv \hat{H}_{\mathrm{int}} - \mathrm{Tr}_E(\hat{H}_{\mathrm{int}}\hat{\rho}_E)\otimes\hat{I}_E - \hat{I}_S \otimes \mathrm{Tr}_S(\hat{H}_{\mathrm{int}}\hat{\rho}_S) + \langle\hat{H}_{\mathrm{int}}\rangle\hat{I}_{SE}$. The standard deviation $\Delta E_{\mathrm{int}}(t) \equiv \sqrt{\langle(\hat{H}^{\mathrm{ent}}_{\mathrm{int}}(t))^2\rangle_t - \langle\hat{H}^{\mathrm{ent}}_{\mathrm{int}}(t)\rangle_t^2}$ is invariant under arbitrary local Hamiltonian shifts (for product states $\hat{\rho}_{SE} = \hat{\rho}_S \otimes \hat{\rho}_E$, $\langle\hat{H}^{\mathrm{ent}}_{\mathrm{int}}\rangle_t = 0$).

2.3 Pure, mixed, and reduced states

1. **Pure states: configuration-field pairs at complete capacity.** Without unmonitored information leakage to an external environment, the system occupies the pure-state level of complete information capacity: relative to any other physical system, its complete objective physical state on M is uniquely determined by a definite real statistical field pair

$(\rho(\cdot,t),[\Phi(\cdot,t)]) \in \mathcal{M}_{\text{ontic}}$—which is not to be interpreted as incomplete knowledge of an underlying classical "true coordinate value"—or, equivalently, by the flat-bundle section $\psi = \sqrt{\rho}e^{i\Phi/\hbar}$ in the Kähler chart of Theorem 1.

2. **Mixed states: ensembles and two-point coherence kernels.** Coupling to an unmonitored heat bath or classical statistical mixing gives an ensemble $\{p_\lambda,(\rho_\lambda,\Phi_\lambda)\}$. The ensemble has an equivalent gauge-invariant representation as a two-point configuration-space coherence density kernel. The diagonal gives the position density, while off-diagonal entries measure residual coherence:

$$\varrho(q,q';t) = \sum_\lambda p_\lambda \sqrt{\rho_\lambda(q,t)}\sqrt{\rho_\lambda(q',t)}e^{i[\Phi_\lambda(q,t)-\Phi_\lambda(q',t)]/\hbar}. \tag{2.2}$$

The total momentum variance combines the intra-branch flow and Fisher localization variances with the classical ensemble dispersion of the branch mean momenta $\bar{p}_\lambda = \int \rho_\lambda \nabla\Phi_\lambda\, d^dq$ around the overall mean $\bar{p} = \sum_\lambda p_\lambda \bar{p}_\lambda$:

$$\Delta_p^2 = \sum_\lambda p_\lambda\, 2m(T_{\text{flow}}^{(\lambda)} + T_{\text{info}}^{(\lambda)}) - |\bar{p}|^2 = \sum_\lambda p_\lambda\Big[(\Delta_p^{(\lambda)})^2 + |\bar{p}_\lambda - \bar{p}|^2\Big].$$

For example, a thermal oscillator in a finite-temperature canonical ensemble has mean phonon occupation $\bar{n} = (e^{\beta\hbar\omega_0}-1)^{-1}$. Equation (2.2) gives

$$\Delta_{q,\beta}^2 = \frac{\hbar}{2m\omega_0}\coth\frac{\beta\hbar\omega_0}{2}, \qquad \Delta_{p,\beta}^2 = \frac{m\hbar\omega_0}{2}\coth\frac{\beta\hbar\omega_0}{2}.$$

Within this single-mode canonical thermal family (where the canonical equilibrium weights are imported from standard statistical mechanics), the Gaussian-reference covariance entropy $S_{\text{state}} = 1 - \ln 2 + \ln(2\bar{n}+1)$ and the von Neumann entropy $S_{\text{vN}} = (\bar{n}+1)\ln(\bar{n}+1) - \bar{n}\ln\bar{n}$ are both strictly increasing functions of $\bar{n}$ and hence of one another.

3. **Reduced states and ensemble-decomposition independence.** Tracing the joint kernel over the environment coordinates gives the subsystem state. The nonselective ensemble obeys the state-independent linear Lindblad–GKSL equation [98, 99] (Theorem 7, part 4). Different pure-state decompositions of the same kernel therefore have identical subsequent macroscopic statistics. Ensemble decompositions cannot carry superluminal signals [92, 93]; see Theorem 11.

Remark (Physical intuition and interpretation: phase memory and thermal decoherence). In a pure state, a single compact information-flow potential $\Phi(q,t)$ extends coherently across configuration space, maintaining a definite phase difference $\Phi(q,t) - \Phi(q',t)$ between any two locations q and q' and enabling full-contrast interference. Coupling to an unmonitored thermal bath scrambles the phases across ensemble branches λ: while the diagonal $\varrho(q,q;t) = \rho(q,t)$ continues to represent the objective spatial density, the off-diagonal modulus $|\varrho(q,q';t)|$ $(q \neq q')$ quantifies residual spatial phase memory across separation $|q-q'|$. As the thermal occupation $\bar{n}$ increases, destructive interference among the branch phase factors $e^{i[\Phi_\lambda(q)-\Phi_\lambda(q')]/\hbar}$ suppresses long-range off-diagonal coherence within a Gaussian envelope and expands the second-moment symplectic area from $\hbar/2$ to $(\hbar/2)\coth(\beta\hbar\omega_0/2)$, capturing the continuous transition from a quantum pure state to a classical thermal mixture.

3 Three fundamental axioms, uniqueness results, and physical interpretation

3.1 Axiom I: objective uncertainty states and topological compactness of the line bundle

Axiom I (kinematic and topological axiom)**.** A microscopic physical system has no classical pointlike "true coordinate value" at any time $t \in \mathbb{R}$ as an objective property of its physical state rather than mere epistemic uncertainty. Without making assumptions regarding internal material constituents, the complete objective physical state of a microscopic system on the configuration manifold M relative to any other physical system is uniquely determined by a pair of real statistical fields

$$(\rho(\cdot,t),[\Phi(\cdot,t)]) \in \mathcal{M}_{\text{ontic}},$$

which characterizes how the system physically manifests rather than representing incomplete knowledge of underlying classical point coordinates. The Ontic Phase Space $\mathcal{M}_{\text{ontic}} = T^*\mathcal{P}_{\text{phys}}(M)/h\mathbb{Z}$ carries the canonical symplectic two-form $\Omega = \int_M \delta\rho(q) \wedge \delta\Phi(q)\, d^d q$.

1. **Objective probability density field and Sobolev regularity.** The field $\rho : M \times \mathbb{R} \to \mathbb{R}_{\geq 0}$ satisfies normalization, $\int_M \rho(q,t)\, d^d q = 1$; finiteness of the second moment, $\int_M |q|^2 \rho(q,t)\, d^d q < \infty$; and Sobolev regularity of the real amplitude, $\sqrt{\rho(\cdot,t)} \in H^1(M)$. The finite Fisher information functional $I_F[\rho] \equiv 4\int_M |\nabla\sqrt{\rho}|^2\, d^d q = \int_{U_t} \rho|\nabla \ln\rho|^2\, d^d q < \infty$ measures the local sharpness of the spatial probability distribution.

2. **Compact information-flow potential and quantized line-bundle holonomy.** On the open set $U_t = \{q \in M \mid \rho(q,t) > 0\}$, the compact information-flow potential field $[\Phi(q,t)]$ is the argument of a section of a flat $U(1)$ line bundle, defined modulo the fundamental action quantum h: $\Phi \sim \Phi + nh$, $n \in \mathbb{Z}$. It satisfies $\sqrt{\rho}\nabla\Phi \in L^2(M)$. The line-bundle holonomy, or circulation, around any closed loop C in U_t obeys the topological quantization law

$$\oint_C \nabla\Phi(q,t) \cdot d\boldsymbol{l} = [n_l + \chi([C])]h, \qquad n_l \in \mathbb{Z}. \tag{3.1}$$

   Here $\chi : \pi_1(M) \to \{0, 1/2\}$ is the flat $\mathbb{Z}_2$ holonomy character associated with order-two torsion homotopy classes of the base manifold, $2[C] = 0$. The trivial or bosonic sector has $\chi = 0$, whereas the spinorial or fermionic torsion sector has $\chi = 1/2$.

3. **Transverse jump law across codimension-1 nodal hypersurfaces.** For a generic complex section $\psi = \psi_R + i\psi_I$, simultaneous vanishing $\psi_R = \psi_I = 0$ defines a codimension-2 vortex locus governed by the loop holonomy law (3.1). Whenever a state possesses a codimension-1 first-order nodal hypersurface $N_t^{(1)} = \{q_0 \in M \mid \rho(q_0,t) = 0\}$ (as occurs for real-phase stationary eigenstates or symmetry-protected nodal walls) across which the signed real amplitude $u \equiv \pm\sqrt{\rho}$ crosses zero linearly in the normal direction $\hat{n}$, the information-flow potential undergoes a $\mathbb{Z}_2$ half-period topological jump across $N_t^{(1)}$, while the normal flow remains smoothly transverse:

$$\lim_{\epsilon\to 0^+} [\Phi(q_0 + \epsilon\hat{n}, t) - \Phi(q_0 - \epsilon\hat{n}, t)] = \frac{2k+1}{2}h = (2k+1)\pi\hbar, \qquad k \in \mathbb{Z}. \tag{3.2}$$

Remark (Physical intuition and interpretation of Axiom I: vortex circulation, standing-wave nodes, and the electromagnetic Aharonov–Bohm effect).

1. **Compactness and the nodal $\pi\hbar$ jump.** In a classical fluid, a single-valued velocity potential $\Phi(q)$ admits only topologically trivial irrotational flow with vanishing circulation around every closed loop. At the microscopic scale, by contrast, $[\Phi]$ takes values in a compact circle fiber of period h (Fig. 1), allowing the potential to wind by an integer multiple $n_l h$ around any

Schematic Summary of Axiom I: Nodal $\mathbb{Z}_2$ Jump vs. Vortex Holonomy

Topological Regime	**(a) Codimension-1 Nodal Hypersurface $N_t^{(1)}$ (Real-Phase / Symmetry-Protected)**	**(b) Codimension-2 Vortex Singularity (Generic Complex Zero Locus)**
Zero-Locus Geometry	Hypersurface $q = q_0$ where $\rho(q_0, t) = 0$ locally partitions U_t into two disconnected half-spaces ($q < q_0$ and $q > q_0$); no closed loop in U_t can encircle $N_t^{(1)}$.	Vortex filament or core $r_\perp = 0$ where $\rho = 0$ has codimension 2 and is encircled by closed loops $C \subset U_t$ in the connected complement.
Real Amplitude & Phase Behavior	The nonnegative root $\sqrt{\rho} = \lvert u\rvert$ retains a V-shaped cusp at q_0, while Φ jumps by half a period, $\Phi(q_0^+) - \Phi(q_0^-) = (2k+1)h/2 = (2k+1)\pi\hbar$ [Eq. (3.2)].	Along a closed loop C encircling the core, the potential rises helically in the $U(1)$ fiber by $\oint_C \nabla\Phi \cdot d\boldsymbol{l} = [n_l + \chi([C])]h$ [Eq. (3.1)].
Physical & Analytic Consequence	Phase factor $e^{i\pi} = -1$ combines with $\sqrt{\rho} = \lvert u\rvert$ so the signed amplitude $u(q) = \pm\sqrt{\rho(q)}$ and section $\psi = ue^{i\Phi_0/\hbar}$ cross zero smoothly, axiomatically excluding Wallstrom's spurious solutions.	Single-valuedness of $\psi = \sqrt{\rho}e^{i\Phi/\hbar}$ quantizes orbital circulation ($n_l\hbar$ for $\chi = 0$) and half-integer spinor holonomy ($(n_l + \frac{1}{2})\hbar$ for $\chi = \frac{1}{2}$).

Figure 1: Schematic comparison of the line-bundle topological conditions in Axiom I. (a) Across a codimension-1 first-order nodal hypersurface $q = q_0$, the nonnegative square root $\sqrt{\rho} = \lvert u\rvert$ retains a V-shaped derivative cusp at zero, whereas the information-flow potential $\Phi(q)$ undergoes the half-period jump $\Delta\Phi = h/2 = \pi\hbar$ ($e^{i\pi} = -1$). Combining $\sqrt{\rho}$ with $e^{i\Phi/\hbar}$ restores smoothness of the signed real amplitude $u(q) = \pm\sqrt{\rho}$ and of $\psi = \sqrt{\rho}e^{i\Phi/\hbar}$ across the node, axiomatically excluding Wallstrom's cusped hydrodynamic solutions. (b) Around a generic codimension-2 vortex singularity, Φ winds helically along a closed loop C in the $U(1)$ fiber by $n_l h$ (or $(n_l + 1/2)h$ in the spinorial torsion sector), quantizing orbital angular momentum and intrinsic spin.

codimension-2 vortex line of vanishing density [Eq. (3.1)] and thereby quantizing orbital angular momentum. Conversely, a one-dimensional oscillator node or a three-dimensional spherical radial nodal shell has codimension 1 and partitions configuration space into disconnected regions that cannot be encircled by a nodeless loop. Across such a first-order nodal wall, Eq. (3.2) mandates a half-period jump $h/2 = \pi\hbar$, matching the smooth zero crossing of the signed real amplitude $u = \pm\sqrt{\rho}$ and encoding signed-amplitude regularity directly into the line-bundle transition structure to exclude cusped spurious solutions [83–85]. In elementary wave-mechanical terms, when a real standing wave $u(q)$ passes linearly through zero from $u > 0$ to $u < 0$ (as in the $2p$ state of hydrogen or the $n = 1$ harmonic oscillator state), its modulus $\sqrt{\rho} = \lvert u\rvert$ folds upward into a V-shaped cusp; without the $\pi\hbar$ phase inversion ($e^{i\pi} = -1$) across the node, the hydrodynamic equations would admit unphysical kinked modes that violate the linear Schrödinger equation.

2. **Minimal electromagnetic coupling and the Aharonov–Bohm topological phase.** For a particle of charge q_e in the four-dimensional electromagnetic gauge potential $A_\mu = (\varphi_e/c, -\boldsymbol{A})$, a gauge-covariant differential replaces the flat differential in the line-bundle connection. The mechanical flow velocity $\boldsymbol{v}(q, t)$ and its circulation satisfy

$$\boldsymbol{v}(q,t) \equiv \frac{1}{m}(\nabla\Phi(q,t) - q_e\boldsymbol{A}(q,t)), \qquad \oint_C m\boldsymbol{v} \cdot d\boldsymbol{l} = n_l h - q_e\Phi_B(C). \tag{3.3}$$

Even when the magnetic field vanishes throughout the region accessible to the particle, $\boldsymbol{B} = 0$, a closed loop enclosing the flux tube Φ_B acquires an observable Aharonov–Bohm interference phase $\Delta\theta_{\mathrm{AB}} = q_e\Phi_B/\hbar$ [100]. The corresponding generalization to an adiabatic parameter space is the Berry geometric phase [101].

3.2 Axiom II: the Bloch-sphere geometric constant and reference information-resolution scale

Axiom II (metric calibration axiom)**.**

1. **Definition of the geometric constant j_0, action parameterization, and postulated reference resolution unit j.** In the quantum Fisher information metric $ds^2_{\mathrm{QFI}} = 4ds^2_{\mathrm{FS}}$, the two-level pure-state projective manifold $\mathbb{CP}^1 \cong S^2$ has total solid-angle area $A_{\mathrm{QFI}}(S^2) = 4\pi$. We define the dimensionless geometric constant j_0 as its reciprocal and postulate the characteristic reference information-resolution unit j:

$$j_0 \equiv \frac{1}{4\pi}, \qquad j = j_0\,\mathrm{nat} = \frac{1}{4\pi}\,\mathrm{nat} \approx 0.079577\,\mathrm{nat}. \tag{3.4}$$

Given the empirical elementary action quantum h, pairing h with j_0 parameterizes the smallest nonzero intrinsic spin projection magnitude (the spin-1/2 scale) $s_{\min} \equiv hj_0 = h/(4\pi) = \hbar/2$ and the reduced Planck constant $\hbar \equiv 2hj_0 = h/(2\pi)$.

2. **Operational correlate of j in continuous monitoring and reference lattice for discrete steps.** Along a continuous diffusive measurement trajectory (Axiom III), extracting one reference unit $\Delta I_{\mathrm{ex}} = j = 1/(4\pi)\,\mathrm{nat}$ corresponds operationally to an expected cumulative quantum-Fisher (Bures) squared-distance displacement $\mathbb{E}[\int_0^t ds^2_{\mathrm{QFI}}] = 2\Delta I_{\mathrm{ex}} = 2j = 1/(2\pi)\,\mathrm{rad}^2$ on projective state space, or a fractional uncertainty compression $\Delta_M(t)/\Delta_M(0) = e^{-j} \approx 0.9235$ for a Gaussian pointer [Eq. (8.1)]. For $k \in \mathbb{N}_0$ reference steps, the cumulative mutual-information lattice is

$$\Delta I^{(k)}_{\mathrm{ex}} = kj = \frac{k}{4\pi}\,\mathrm{nat}, \qquad k \in \mathbb{N}_0 = \{0, 1, 2, \ldots\}. \tag{3.5}$$

In the baseline Markovian dynamics of Axiom III, cumulative mutual information $I_{\mathrm{ex}}(t)$ varies continuously from zero (so a weak measurement or a highly asymmetric binary prior with $H_b(p) < j$ exchanges less than j), and $\Delta I^{(1)}_{\mathrm{ex}} = j$ serves as a reference unit rather than a hard floor; whether microscopic channels additionally exhibit non-Markovian metastability barriers or commitment gates at integer multiples kj is isolated as the independent exploratory hypothesis Assumption H3 (Section 3.4).

Remark (Physical intuition and interpretation of Axiom II: geometric parameterization of $\hbar$ and $\hbar/2$). The factor 2π relating h and $\hbar = h/(2\pi)$, and the factor 4π in the spin-1/2 projection $\hbar/2 = h/(4\pi)$, reflect the geometry of $U(1)$ phase cycles and $SU(2)/U(1) \cong S^2$ projective state space. Rather than claiming an a priori derivation of the empirical constant h or of representation-theoretic spin from pure geometry alone, Axiom II organizes these factors around a single geometric ratio: the reciprocal $j_0 = 1/(4\pi)$ of the total quantum-Fisher solid angle $A_{\mathrm{QFI}}(S^2) = 4\pi$, which represents the fraction of one steradian on the Bloch sphere. Pairing one unit solid-angle fraction j_0 with the fundamental action period h associates the spin-1/2 angular-momentum scale $s_{\min} = hj_0 = \hbar/2$ with a single elementary cell and the orbital quantum $\hbar = 2hj_0 = h/(2\pi)$ with a paired cycle. Note also that $j = 1/(4\pi)\,\mathrm{nat} \approx 0.0796\,\mathrm{nat}$ represents an elementary resolution step, whereas complete Shannon discrimination between two equally likely orthogonal states requires $\ln 2 \approx 0.6931\,\mathrm{nat} \approx 8.71\,j$.

3.3 Axiom III: finite-bandwidth action–information dynamics

Axiom III (dynamical axiom)**.** Consider a system of mass $m > 0$ in an external real scalar potential $V(q,t)$, coupled to an external diffusive measurement channel with pure entanglement interaction energy standard deviation $\Delta E_{\rm int}(t) \geq 0$ [Eq. (8.4)]. To preserve the Sobolev H^1 amplitude domain of Axiom I, the monitored configuration observable $M(q)$ is required either to be a smooth function $M \in C^1(M)$ with $\int(|M(q)|^4 + |\nabla M(q)|^2)\rho(q,t)\,d^dq < \infty$ (for continuous-variable monitoring) or a piecewise-constant quantum nondemolition (QND) observable $M(q) = \sum_{k=1}^N m_k \mathbf{1}_{\Omega_k}(q)$ on disjoint branch supports Ω_k satisfying Assumption H2(a). Its current sample mean and variance are $\overline{M}_t \equiv \int M(q)\rho(q,t)\,d^dq$ and $\Delta_M^2(t) \equiv \int(M(q) - \overline{M}_t)^2\rho(q,t)\,d^dq$, respectively.

1. **Total energy functional and constitutive law for the second momentum moment.** In the nonrelativistic limit, characteristic lengths greatly exceed the reduced Compton wavelength $\widetilde{\lambda}_C = \hbar/(mc)$. The expected total energy then comprises the macroscopic information-flow kinetic energy $T_{\rm flow}$, the Fisher information kinetic energy $T_{\rm info}$ required to maintain spatial localization, and the external potential energy. The Fisher form is selected within the local class specified in Proposition 3.1, including its pointwise subsystem-composition condition. Its coefficient $(hj_0)^2/(2m) = \hbar^2/(8m)$ is a constitutive normalization of this axiom:

$$H[\rho,\Phi] = \int_{\mathbb{R}^d} \rho(q,t)\left[\frac{|\nabla\Phi(q,t)|^2}{2m} + \frac{(hj_0)^2}{2m}|\nabla\ln\rho(q,t)|^2 + V(q,t)\right]d^dq. \tag{3.6}$$

   The expected total kinetic energy provides the constitutive definition of the total second momentum moment: $\langle p^2\rangle \equiv 2m(T_{\rm flow} + T_{\rm info}) = \int_{\mathbb{R}^d}\rho[|\nabla\Phi|^2 + (hj_0)^2|\nabla\ln\rho|^2]d^dq$ (where the identity $\nabla\ln\rho = 2\nabla\sqrt{\rho}/\sqrt{\rho}$ converts the Fisher prefactor $(hj_0)^2/(2m) = \hbar^2/(8m)$ into the standard Schrödinger amplitude-gradient kinetic energy $\int_{\mathbb{R}^d}[\hbar^2/(2m)]|\nabla\sqrt{\rho}|^2\,d^dq$).

2. **Nyquist–Planck maximum channel capacity law and additive composition across elementary channels.** An *elementary diffusive measurement channel* α is an indecomposable system–probe coupling generating an independent scalar Wiener readout innovation $dW_t^{(\alpha)}$ with pure entanglement interaction energy spread $\Delta E_{{\rm int},\alpha}(t)$. Each elementary diffusive channel extracts classical mutual information $I_{{\rm ex},\alpha}(t)$ at an instantaneous rate $\gamma_{{\rm info},\alpha}(t) \equiv dI_{{\rm ex},\alpha}(t)/dt$ (in nat/s) bounded by

$$0 \leq \gamma_{\rm info}(t) \equiv \frac{dI_{\rm ex}}{dt} \leq C_{\max}(t) \equiv \frac{\Delta E_{\rm int}(t)}{h}\ \text{nat/s}. \tag{3.7}$$

   For K mutually independent elementary channels $\{\alpha = 1,\ldots,K\}$ with independent readout records $\{dW_t^{(\alpha)}\}$ and interaction Hamiltonian $\hat{H}_{\rm int} = \sum_{\alpha=1}^K \hat{H}_{{\rm int},\alpha}$, both the information-extraction rate and the capacity ceiling compose additively (ℓ_1 resource accounting): $\gamma_{\rm info}^{\rm tot}(t) = \sum_{\alpha=1}^K \gamma_{{\rm info},\alpha}(t) \leq C_{\max}^{\rm tot}(t) \equiv \sum_{\alpha=1}^K \Delta E_{{\rm int},\alpha}(t)/h$. This elementary-channel ℓ_1 composition rule remains strictly invariant when N independent copies of a system–probe apparatus are grouped together (avoiding the $\sqrt{N}$ quadrature paradox that would arise if Eq. (3.7) were applied to the global ℓ_2 standard deviation $\sqrt{\sum_\alpha \Delta E_{{\rm int},\alpha}^2}$).

3. **Unified stochastic action–information evolution equations.** On the support $\{\Delta_M(t) > 0\}$, the objective probability density field $\rho(q,t)$ and compact information-flow potential field $\Phi(q,t)$ obey the unified Itô stochastic partial differential equations below. At $\Delta_M(t) = 0$, the stochastic measurement term vanishes automatically, and the system reaches an eigenstate absorbing boundary. Without measurement coupling, $\gamma_{\rm info} = 0$, these equations are equivalent

to stationary variation of the canonical symplectic action $S[\rho,\Phi]=\int(\int_M \Phi\partial_t\rho\, d^dq - H[\rho,\Phi])dt$:

$$\begin{aligned} d\rho(q,t) = &-\nabla\cdot\left(\rho(q,t)\frac{\nabla\Phi(q,t)}{m}\right)dt \\ &+\sqrt{2\gamma_{\text{info}}(t)}\left(\frac{M(q)-\overline{M}_t}{\Delta_M(t)}\right)\rho(q,t)\,dW_t, \end{aligned} \tag{3.8a}$$

$$d\Phi(q,t) = -\frac{\delta H[\rho,\Phi]}{\delta\rho(q,t)}dt + d\Phi_{\text{back}}(q,t). \tag{3.8b}$$

Here dW_t is the increment of a standard Wiener process on the complete filtered probability space $(\Omega_{\text{prob}}, F_t, \mathbb{P})$, with $\mathbb{E}[dW_t]=0$ and $(dW_t)^2=dt$. For a pure configuration measurement with no additional phase shift, the conjugate phase-backaction term vanishes identically: $d\Phi_{\text{back}}(q,t)\equiv 0$.

4. **Total energy balance during an open measurement.** For a time-independent external potential $V(q)$ and a pure configuration measurement ($d\Phi_{\text{back}}\equiv 0$), the deterministic Hamiltonian drift conserves $H[\rho,\Phi]$ identically ($\frac{(hj_0)^2}{2m}d\mathbb{E}[I_F[\rho]]_{\text{det}} + d\mathbb{E}[T_{\text{flow}}+V]_{\text{det}}=0$), while T_{flow} and V are linear in ρ and therefore have zero second-order Itô correction ($d\mathbb{E}[T_{\text{flow}}+V]_{\text{meas}}=0$). Consequently, the net Itô increment in expected total system energy equals the pure measurement Fisher-compression work dE_{channel} injected by the channel [Theorem 11, Eq. (7.10)]:

$$d\mathbb{E}[H[\rho,\Phi]] = \frac{(hj_0)^2}{2m}d\mathbb{E}[I_F[\rho]] + d\mathbb{E}[T_{\text{flow}}+V] = \frac{(hj_0)^2}{2m}d\mathbb{E}[I_F[\rho]]_{\text{meas}} = dE_{\text{channel}}. \tag{3.9}$$

Remark (Physical intuition and interpretation of Axiom III: continuous Bayesian information locking and the hierarchy of coupling rates).

1. **Physical mechanism of the stochastic diffusion term in Eq.** (3.8a)**.** The first term, $-\nabla\cdot(\rho\nabla\Phi/m)dt$, is conservative continuity transport. The second, $\sqrt{2\gamma_{\text{info}}(t)}[(M(q)-\overline{M}_t)/\Delta_M(t)]\rho(q,t)dW_t$, is the channel's real-time information-update force. A positive readout fluctuation, $dW_t>0$, simultaneously increases the density $\rho(q,t)$ wherever $M(q)>\overline{M}_t$ and decreases it wherever $M(q)<\overline{M}_t$. For $dW_t<0$, the update instead concentrates weight in the lower-value regions. Since $\int(M(q)-\overline{M}_t)\rho(q,t)\,d^dq\equiv 0$, each instantaneous stochastic update exactly conserves total probability, $\int\rho\,d^dq=1$, without requiring subsequent renormalization.
2. **Constitutive hierarchy of the bare coupling rate $\kappa_{\text{meas}}(t)$ and the state-dependent information-extraction rate $\gamma_{\text{info}}(t)$.** The laser power or microwave readout pulse envelope alone sets the deterministic bare coupling rate $\kappa_{\text{meas}}(t)\geq 0$. The instantaneous information-extraction rate follows as a derived physical quantity:

$$\gamma_{\text{info}}(t) = 2\kappa_{\text{meas}}(t)\Delta_M^2(t) \leq C_{\max} = \frac{\Delta E_{\text{int}}}{h}. \tag{3.10}$$

Before collapse, a large variance $\Delta_M^2(t)$ allows the probe to distinguish different positions efficiently. The rate $\gamma_{\text{info}}(t)$ is correspondingly high, subject to the upper bound $C_{\max}$. As the wave packet contracts toward a single eigenvalue, $\Delta_M^2(t)\to 0$, no uncertainty remains to be extracted, so $\gamma_{\text{info}}(t)$ smoothly tends to zero. Substitution of Eq. (3.10) into Eq. (3.8a) removes the denominator $\Delta_M(t)$. In the nonselective ensemble average, with unread records, all nonlinear terms involving the state mean $\overline{M}_t$ then cancel pointwise under the second-order Itô product rule. The macroscopic ensemble therefore obeys a linear Lindblad master equation, as established in Theorem 7.
3. **Multichannel phase backaction and independence of Lorentz slicing.** For K local phase-quadrature channels with deterministic apparatus-controlled bare phase-coupling rates

$\kappa_{\mathrm{phase},\alpha}(t) \geq 0$ and state-dependent phase-information rates $\gamma_{\mathrm{phase},\alpha}(t) = 2\kappa_{\mathrm{phase},\alpha}(t)\Delta^2_{M_\alpha}(t)$ [paralleling Eq. (3.10)], the conjugate phase-backaction term is

$$\begin{aligned} d\Phi_{\mathrm{back}}(q,t) &= \hbar \sum_{\alpha=1}^{K} \sqrt{2\gamma_{\mathrm{phase},\alpha}(t)} \left(\frac{M_\alpha(q) - \overline{M}_{\alpha,t}}{\Delta_{M_\alpha}(t)} \right) d\widetilde{W}_t^{(\alpha)} \\ &= 2\hbar \sum_{\alpha=1}^{K} \sqrt{\kappa_{\mathrm{phase},\alpha}(t)} (M_\alpha(q) - \overline{M}_{\alpha,t}) d\widetilde{W}_t^{(\alpha)}. \end{aligned} \tag{3.11}$$

Physically, whereas the density innovation in Eq. (3.8a) directly reweights spatial probability without shifting the phase, Eq. (3.11) describes conjugate phase-quadrature backaction: each Wiener increment $d\widetilde{W}_t^{(\alpha)}$ imparts a random spatial phase tilt $d\Phi_{\mathrm{back}} \propto (M_\alpha(q) - \overline{M}_{\alpha,t}) d\widetilde{W}_t^{(\alpha)}$—and hence a random momentum kick $\nabla(d\Phi_{\mathrm{back}}) \propto \nabla M_\alpha(q)\, d\widetilde{W}_t^{(\alpha)}$—that leaves the instantaneous density $\rho(q,t)$ unchanged while scrambling relative phases between distinct positions $q \neq q'$. Under the second-order Itô expansion of $\psi(q,t)\psi^*(q',t)$, the trajectory means $\overline{M}_{\alpha,t}$ cancel in $M_\alpha(q) - M_\alpha(q')$ and the denominators $\Delta_{M_\alpha}(t)$ cancel via $\gamma_{\mathrm{phase},\alpha} = 2\kappa_{\mathrm{phase},\alpha}\Delta^2_{M_\alpha}$, yielding the state-independent linear Lindblad dephasing superoperator $-2\sum_\alpha \kappa_{\mathrm{phase},\alpha}(t)[M_\alpha(q) - M_\alpha(q')]^2 \varrho(q,q';t)$. For two spacelike-separated local measurement regions $R_A \sim R_B$, the independent Wiener processes are orthogonal and the local Lindblad superoperators commute, $[L_A, L_B] = 0$, rendering ensemble measurement statistics independent of the ordering of spacelike-separated updates.

Proposition 3.1 (Conditional uniqueness of the Fisher localization kinetic energy and quantum Cramér–Rao geometric calibration of the channel capacity)**.**

1. **Conditional uniqueness of the Fisher information functional** [72, 74, 102–104]. For a fixed mass parameter m, let the first-gradient localization kinetic energy be $T_{\mathrm{info}}[\rho] = \int_{\mathbb{R}^d} \rho(q) f_d(\rho(q), \nabla \ln \rho(q))\, d^d q$, with a continuous, nonnegative local cost. Suppose that it satisfies (i) invariance under Euclidean translations and rotations; (ii) strict energy additivity for independent subsystems, $T_{\mathrm{info}}[\rho_{AB}] = T_{\mathrm{info}}[\rho_A] + T_{\mathrm{info}}[\rho_B]$ for every statistically independent composite state $\rho_{AB}(q_A, q_B) = \rho_A(q_A)\rho_B(q_B)$; and (iii) homogeneous spatial scale covariance, $T_{\mathrm{info}}[\rho_\lambda] = \lambda^2 T_{\mathrm{info}}[\rho]$ under $q \mapsto \lambda^{-1} q$, $\rho_\lambda(q) = \lambda^d \rho(\lambda q)$. In addition, impose the following local composition condition, which is stronger than integrated energy additivity: in independent Euclidean product coordinates the local costs add pointwise, and a locally uniform density has zero localization cost. Writing the rotationally invariant cost as $f_d(r, w)$, with $r > 0$ and $w = |\nabla \ln \rho|^2 \geq 0$, these requirements are

$$\begin{aligned} f_{d_A+d_B}(r_A r_B, w_A + w_B) &= f_{d_A}(r_A, w_A) + f_{d_B}(r_B, w_B), \\ f_d(r, 0) &= 0. \end{aligned}$$

The cost is then $f_d(r, w) = c_0 w$, where the same constant $c_0 \geq 0$ applies to these coordinates. The structural assumptions determine this form up to c_0; the constitutive normalization in Axiom III fixes $c_0 = (\hbar j_0)^2/(2m) = \hbar^2/(8m)$. Thus, with the stated local composition and normalization conditions,

$$\begin{aligned} &\text{local composition, zero-gradient cost, continuity, and Axiom III} \\ &\qquad\Longrightarrow \qquad T_{\mathrm{info}}[\rho] = \frac{(\hbar j_0)^2}{2m} I_F[\rho]. \end{aligned} \tag{3.12}$$

2. **Quantum Cramér–Rao speed limit, geometric calibration of $C_{\max} = \Delta E_{\mathrm{int}}/\hbar$, and distinction between coherent unitary gates and diffusive readout.** Consider the joint evolution of a system S and probe E generated by the pure entanglement interaction $\hat{H}_{\mathrm{int}}^{\mathrm{ent}}$.

Along distinguishable directions in projective state space, the instantaneous speed in quantum Fisher information (Bures) distance obeys the Braunstein–Caves / Mandelstam–Tamm upper bound $ds_{\mathrm{QFI}}/dt \le 2\Delta E_{\mathrm{int}}/\hbar = 4\pi\Delta E_{\mathrm{int}}/h$ [86, 90, 97]. Because one-dimensional Bures arc length s_{QFI} and classical Shannon mutual information I_{ex} belong to distinct operational categories (under continuous diffusive monitoring in Eq. (3.8a), the infinitesimal state displacement is driven by $dW_t \sim \sqrt{dt}$, so the expected squared Bures displacement obeys the quadratic Itô scaling $\mathbb{E}[ds^2_{\mathrm{QFI}}] = 4\mathbb{E}[\int(\delta\rho)^2/(4\rho)\, d^dq] = 2\gamma_{\mathrm{info}}\, dt$, whereas under coherent unitary flow ds_{QFI} grows linearly in dt), the capacity bound Eq. (3.7) is an independent physical postulate of Axiom III rather than a deductive identity of spherical geometry. Calibrating the maximum diffusive information-extraction rate by pairing the maximal quantum-Fisher angular speed $(ds_{\mathrm{QFI}}/dt)_{\max}$ with the reference unit $j = 1/(4\pi)$ nat of Axiom II yields the Nyquist–Planck rate scale:

$$\begin{aligned} &\frac{ds_{\mathrm{QFI}}}{dt} \le \frac{2\Delta E_{\mathrm{int}}}{\hbar} = \frac{4\pi\Delta E_{\mathrm{int}}}{h} \\ \Longrightarrow \qquad &C_{\max} \equiv j\left(\frac{ds_{\mathrm{QFI}}}{dt}\right)_{\max} = \frac{1}{4\pi}\cdot\frac{4\pi\Delta E_{\mathrm{int}}}{h} = \frac{\Delta E_{\mathrm{int}}}{h}\ \text{nat/s}. \end{aligned} \tag{3.13}$$

We emphasize the physical domain of Eq. (3.7): it governs *continuous diffusive measurement channels* generating a classical Markovian record $\mathcal{F}_t$ [Eqs. (3.8a) and (5.34)], as distinct from *closed coherent unitary premeasurement gates* $\hat{U}_{\mathrm{pre}}$ [Theorem 2(3)]. For example, under a coherent two-qubit entangling gate $\hat{H}_{\mathrm{int}} = \hbar g\, \hat{\sigma}_z \otimes \hat{\sigma}_x$ with initial state $((|0\rangle + |1\rangle)/\sqrt{2}) \otimes |0\rangle$ ($\Delta E_{\mathrm{int}} = \hbar g$), unitary evolution rotates the conditional probe states to orthogonality at the Mandelstam–Tamm time $\tau_{\mathrm{MT}} = \pi/(4g)$, generating $\ln 2 \approx 0.693$ nat of coherent quantum entanglement while $(\Delta E_{\mathrm{int}}/h)\tau_{\mathrm{MT}} = 1/8$ nat. However, that closed unitary gate produces no irreversible classical readout record; converting the correlated probe state into a classical record still requires coupling the probe to an open diffusive amplification/readout channel subject to Axiom III. Likewise, while a QND measurement ($\nabla M = 0$ on each support Ω_k) injects zero spatial Fisher backaction energy into the measured observable itself [Eq. (7.10)], ΔE_{int} acts as a dynamical bandwidth resource setting the rate of interbranch phase randomization rather than a dissipative heat sink. Whereas Landauer's principle [22] sets the minimum thermodynamic heat $k_B T \ln 2$ dissipated during irreversible bit erasure, neither Shannon's coding theorem [17] nor Holevo's bound [11] fixes a Hamiltonian rate limit; Eq. (3.13) thus remains an independent, testable dynamical postulate of Axiom III (Section 8.1).

Proof. **Step 1: Fisher localization kinetic energy.** Euclidean rotational invariance permits f_d to depend only on the scalar ρ and the squared gradient magnitude $w \equiv |\nabla \ln \rho|^2$. Under $\rho_\lambda(q) = \lambda^d \rho(\lambda q)$, one has $\nabla_q \ln \rho_\lambda(q) = \lambda(\nabla \ln \rho)(\lambda q)$ and therefore $w_\lambda = \lambda^2 w$. Scale covariance gives

$$\int \rho(y) f_d(\lambda^d \rho(y), \lambda^2 w(y))\, d^d y = \lambda^2 \int \rho(y) f_d(\rho(y), w(y))\, d^d y.$$

To establish independence from ρ and linearity in w, we invoke the pointwise subsystem-composition condition: taking two subsystems of equal dimension and localizing the nonzero gradient in either subsystem yields

$$\begin{aligned} f_{2d}(r_A r_B, w) &= f_d(r_A, w) + f_d(r_B, 0) = f_d(r_A, w), \\ f_{2d}(r_A r_B, w) &= f_d(r_A, 0) + f_d(r_B, w) = f_d(r_B, w). \end{aligned}$$

Since r_A and r_B are arbitrary positive local density values, $f_d(r, w)$ is independent of r. Applying the same argument to dimensions d_A and d_B shows that the remaining function $F(w)$ is common

to the dimensions in the composition rule. Consequently,

$$F(w_A + w_B) = F(w_A) + F(w_B), \qquad F(0) = 0.$$

For any reference value $w_0 > 0$ and nonnegative rational α, repeated addition gives $F(\alpha w_0) = \alpha F(w_0)$. Continuity extends this identity to every real $\alpha \geq 0$, so $F(w) = c_0 w$ with $c_0 = F(w_0)/w_0$; nonnegativity gives $c_0 \geq 0$. For the independent product state $\rho_{AB} = \rho_A \rho_B$, the identity

$$|\nabla_{A,B} \ln(\rho_A \rho_B)|^2 = |\nabla_A \ln \rho_A|^2 + |\nabla_B \ln \rho_B|^2$$

and the unit normalization of each marginal yield strict integrated additivity, $T_{\mathrm{info}}[\rho_{AB}] = T_{\mathrm{info}}[\rho_A] + T_{\mathrm{info}}[\rho_B]$. The resulting functional also obeys the required scale law because $w_\lambda = \lambda^2 w$. Whereas nonquadratic powers $|\nabla \ln \rho|^p$ $(p \neq 2)$ violate pointwise subsystem additivity, the quadratic Fisher functional is uniquely selected within this local class, with its overall prefactor $(\hbar j_0)^2/(2m) = \hbar^2/(8m)$ fixed by the second-momentum constitutive law of Axiom III [Eq. (3.6)] and the action scale $\hbar j_0 = \hbar/2$ of Axiom II (generalizing to $(\hbar j_0)^2/(2m_a)$ for subsystems of distinct masses m_a).

Step 2: geometric calibration of the maximum channel capacity. By Theorem 1 in Section 4, Eq. (4.3), the rescaled Fisher–Rao norm of a reduced horizontal tangent vector (orthogonal to global phase shifts) satisfies $ds^2_{\mathrm{QFI}} = (2/\hbar) G_{\mathrm{FR}}(\delta\rho, \delta\Phi) = 4 \int |\delta\psi_\perp|^2 d^d q$. Under $\hat{H}^{\mathrm{ent}}_{\mathrm{int}}$, the state-space velocity orthogonal to the current ray has norm $\|\delta\psi_\perp/dt\| = \Delta E_{\mathrm{int}}/\hbar$, where $\Delta E_{\mathrm{int}} = \sqrt{\langle (\hat{H}^{\mathrm{ent}}_{\mathrm{int}})^2 \rangle - \langle \hat{H}^{\mathrm{ent}}_{\mathrm{int}} \rangle^2}$, giving the Braunstein–Caves speed limit $ds_{\mathrm{QFI}}/dt \leq 2\Delta E_{\mathrm{int}}/\hbar$ [97]. Pairing this maximal state-space speed with the reference quantum $j = 1/(4\pi)$ nat of Axiom II, Eq. (3.4), and substituting $2\pi\hbar = h$ gives the constitutive calibration in Eq. (3.13). □

3.4 Separate mathematical technical conditions and exploratory physical assumptions H1–H4

To keep the logical architecture transparent, we separate the two mathematical-physics domain and measurement conditions (Assumptions H1–H2) invoked in Theorems 1–7 from the two exploratory physical hypotheses (Assumptions H3–H4) used exclusively in Prediction II (Section 8.2). Table 4 summarizes the logical status of all inputs, axioms, calibrations, and hypotheses.

Assumption H1 (Joint Sobolev regularity of amplitude and phase and decay at spatial infinity)**.** Throughout the time interval under consideration, the signed real amplitude satisfies $u(\cdot, t) \in H^2_{\mathrm{loc}}(M) \cap H^1(M)$, the local phase representative on $U_t = \{q \in M \mid \rho(q,t) > 0\}$ satisfies $\nabla\Phi(\cdot, t) \in H^1_{\mathrm{loc}}(U_t)$ such that the complex section $\psi = \sqrt{\rho} e^{i\Phi/\hbar}$ belongs to the Hamiltonian operator domain $H^2_{\mathrm{loc}}(M, \mathbb{C}) \cap H^1(M, \mathbb{C})$ with $\int_M (1 + |q|^2 + |\nabla\Phi|^2)\rho(q,t)\, d^d q < \infty$, and the boundary decay conditions $\lim_{|q|\to\infty} |q|\rho(q,t) = 0$ and $\lim_{|q|\to\infty} \rho(q,t)|\nabla\Phi(q,t)| = 0$ hold.

Physical motivation and consequences. Requiring $\sqrt{\rho} \in H^1(M)$ alone is insufficient for classical or strong Schrödinger differentiability if the phase Φ has gradient cusps on U_t (for example, $\rho(x) = (\sqrt{\pi}a)^{-1} e^{-x^2/a^2}$ with $\Phi(x) = \hbar|x|/a$ has smooth $\sqrt{\rho}$ and finite $\int \rho(\partial_x \Phi)^2 dx$, but $\psi = \sqrt{\rho} e^{i|x|/a}$ has a derivative jump at $x = 0$ and a distributional delta term in $\partial_x^2 \psi$). Requiring $\psi \in H^2_{\mathrm{loc}}(M, \mathbb{C}) \cap H^1(M, \mathbb{C})$ ensures both finite total kinetic energy and local second-order differentiability of the coupled Madelung–Schrödinger system, while the decay conditions eliminate boundary flux terms at infinity.

Assumption H2 (Quantum nondemolition branch-flow isolation or dominance of a strong measurement pulse)**.** During the open measurements considered in Theorem 7 and Prediction I, at least one of the following conditions holds: (a) *branch-flow isolation*, the strict quantum nondemolition (QND) condition, in which the measured observable has a constant value m_k on each of the disjoint eigenvalue supports Ω_k, with $\nabla M(q) = 0$ in their interiors, and the normal probability flux through every support boundary vanishes; or (b) the *strong-pulse measurement-dominated limit*,

in which the channel information-extraction rate greatly exceeds the free Hamiltonian drift rate during the measurement window. The complete three-component covariance dynamics and the breathing–dissipation stationary solution for a non-QND continuous position measurement in a general external potential are given in Section 8.1, Eq. (8.3).

Physical motivation and consequences. Uncompensated Hamiltonian transport across the channel supports Ω_k would remix eigenbranches during readout; condition H2(a) eliminates this cross-boundary flux exactly (as in dispersive superconducting-qubit readout with $[\hat{H}_S, \hat{\sigma}_z] = 0$), whereas H2(b) renders it negligible over a short, strong readout pulse so that extracted information goes directly into reducing observable uncertainty. Consequently, the measurement-only branch equations hold exactly under H2(a) and represent the leading measurement-dominated approximation under H2(b) (becoming exact for the first two moments whenever Hamiltonian drift contributions to those moments vanish).

Assumption H3 (Exploratory non-Markovian subordination to a discrete information clock along monitored trajectories; used only in Prediction II)**.** Beyond the baseline time-local Markovian SDE of Eq. (3.8a), suppose that microscopic channel mutual-information extraction is governed by a non-Markovian subordination mechanism or metastable information barriers with period $j = 1/(4\pi)$ nat: (i) in the weak-barrier regime, the effective extraction rate is periodically modulated as $\gamma_{\rm eff}(I_{\rm ex}) = \gamma_0[1 - \alpha_b \cos(2\pi I_{\rm ex}/j)]$ $(0 < \alpha_b < 1)$, producing residence-time extrema at the lattice points $I_{\rm ex}(t) = kj$; or (ii) in the strong-threshold (hard-gate) regime, irreversible macroscopic record commitment is suppressed until the accumulated channel information reaches the first quantum threshold $\Delta I_{\rm ex} = j$ at pulse duration $\tau_{\rm knee} = \Delta t_{\rm min}(j) = \hbar/(2\Delta E_{\rm int})$.

Physical motivation and consequences. In the baseline Markovian equations (3.8a) and (5.33), any nonzero coupling $\kappa_{\rm meas} > 0$ produces immediate linear ensemble decoherence $1 - V(t) \propto \kappa_{\rm meas} t$ for arbitrarily short times $t > 0$. Assumption H3 explicitly isolates the non-Markovian hypothesis required if the reference quantum $j = 1/(4\pi)$ nat of Axiom II acts as a physical dynamical barrier—analogous to Josephson washboard potentials or single-electron Coulomb blockade—generating either periodic histogram residence peaks or a short-pulse commitment threshold $\tau_{\rm knee}$ [Section 8.2, Eqs. (8.6)–(8.8)].

Assumption H4 (Exploratory equal conversion of leaked channel information into covariance entropy under unmonitored weak measurement backaction; used only in Prediction II(2))**.** When an oscillator prepared in its minimum-uncertainty ground state is subjected to an unmonitored weak measurement backaction or phase-diffusion channel (or a weakly coupled finite-temperature environment) that extracts discrete mutual-information quanta $\Delta I_{\rm ex} = kj$ without record conditioning, the unmonitored backaction increases the system's Gaussian-reference covariance differential entropy by $\Delta S_{\rm state}[\Sigma] = \Delta I_{\rm ex} = kj$.

Physical motivation and consequences. A ground-state oscillator coupled solely to a passive zero-temperature amplitude-damping bath in the rotating-wave approximation remains in its ground state without secular entropy growth. In contrast, an active unmonitored measurement channel (such as unread homodyne radiation or dispersive dephasing with non-QND backaction) injects phase-space noise into the unread ensemble. Assumption H4 is an exploratory phenomenological hypothesis positing that each unmonitored resolution step j expands the logarithmic symplectic area by $\Delta S_{\rm state}[\Sigma] = j$, yielding the discrete symplectic-area lattice $D_{\Sigma,k} = (\hbar/2)\exp(k/4\pi)$ in Section 8.2, Eq. (8.7).

Table 4: Logical classification of empirical inputs, axioms, calibrations, technical domain conditions, and exploratory hypotheses in QICM.

Element	Logical Status	Primary Location	Role and Scope in the Framework
h, m, e, c	Empirical inputs	Sec. 2.2, Table 3	Elementary action period h, particle mass m, charge e, and light speed c imported from experiment.
$(\rho, [\Phi]) \in \mathcal{M}_{\text{ontic}}$, $I_F[\rho] < \infty$, Eqs. (3.1)–(3.2)	Axiom I (kinematic & topological postulate)	Sec. 3.1	Defines the ontic state space, finite Fisher regularity, quantized vortex holonomy, and the $\pi\hbar$ jump across codimension-1 nodes (axiomatically closing Wallstrom's objection).
$j_0 \equiv 1/(4\pi)$, $s_{\min} = hj_0 = \hbar/2$, $\hbar = 2hj_0$, $j = j_0$ nat	Axiom II (definition, parameterization & reference scale)	Sec. 2.1, Sec. 3.2, Eqs. (3.4)–(3.5)	Defines j_0 from $A_{\text{QFI}}(S^2) = 4\pi$, parameterizes $\hbar/2$ and $\hbar$ with h, and postulates $j = 1/(4\pi)$ nat as a reference resolution step.
$T_{\text{info}} = \frac{\hbar^2}{8m} I_F[\rho]$	Axiom III(1) & Prop. 3.1(1) (conditional uniqueness + normalization)	Sec. 3.3, Eqs. (3.6), (3.12)	Quadratic Fisher form uniquely selected under Euclidean invariance, scale covariance, and pointwise subsystem composition; prefactor $\hbar^2/(8m)$ fixed by constitutive normalization.
$C_{\max,\alpha} = \Delta E_{\text{int},\alpha}/h$	Axiom III(2) & Prop. 3.1(2) (dynamical postulate + QFI calibration)	Sec. 3.3, Eqs. (3.7), (3.13); Sec. 8.1	Postulates an upper bound on the mutual-information rate of an elementary diffusive channel α, calibrated via $j \cdot (ds_{\text{QFI}}/dt)_{\max}$ and tested in Prediction I.
Coupled Itô PDEs	Axiom III(3) (dynamical postulate)	Sec. 3.3, Eqs. (3.8a)–(3.8b)	Unifies closed Madelung flow ($\gamma_{\text{info}} = 0$) with diffusive configuration measurement ($d\Phi_{\text{back}} = 0$).
Assumptions H1–H2	Technical domain & measurement conditions	Sec. 3.4	H1 specifies $\psi \in H^2_{\text{loc}} \cap H^1$ regularity and decay; H2 specifies QND branch isolation (H2a) or strong-pulse dominance (H2b).
Postulate S, envariance, and Eq. (3.14)	Probability semantics & conditional H-theorem	Sec. 3.5	Identifies ρ as objective propensity density and proves coarse-grained KL relaxation under an explicit advection–diffusion and coercivity closure.
Assumptions H3–H4	Exploratory physical hypotheses	Sec. 3.4, Sec. 8.2	Non-Markovian j-clock subordination (H3) and unmonitored backaction entropy conversion (H4), used *solely* in Prediction II.
Spin–statistics link	Explicit compatibility assumption	Sec. 7.2, Thm. 9(2)	Assumes $\Delta\Phi_{ab} \equiv \Delta\Phi_{2\pi} \pmod h$ on the framed point-particle quotient M_N.
DBI parameter $\xi_c = 1/8$	Phenomenological UV calibration	Sec. 7.3, Thm. 10, Eqs. (7.6)–(7.9)	Calibrates single-particle nodal osmotic-energy saturation to the pair rest-energy scale $2mc^2$.

3.5 Probability semantics, coarse-grained relaxation to quantum equilibrium, and envariance

Postulate S (Objective propensity probabilities and quantum equilibrium, with two dynamical origins)**.** The objective density field $\rho(q,t)$ is the system's objective propensity distribution. For measurements satisfying H2, the initial integral measure carried by each eigenvalue support Ω_k, $P_k(0) = \int_{\Omega_k} \rho(q,0)\, d^d q$, defines its objective prior weight. Global normalization, $\int_M \rho(q,t)\, d^d q = 1$, expresses conservation of total probability charge. Under the branch-isolation and asymptotic-resolution conditions stated in Theorem 7 in Section 5, the stochastic information-update dynamics of Axiom III convert this initial prior measure $P_k(0)$ into the frequency with which individual

measurements collapse to branch k. The bounded-martingale argument is exact for the isolated branch dynamics; the strong-pulse condition H2(b) gives its measurement-dominated approximation.

Envariance and the conditional coarse-grained H-theorem. Envariance constrains symmetry-related prior weights, whereas relaxation concerns the relation between a sampling distribution and the prescribed objective density. Here "objective propensity" means the single-case physical measure $P_\Omega(t) = \int_\Omega \rho(q,t)\, d^d q$ assigned by the ontic state $(\rho, [\Phi])$ to each measurable region $\Omega \subset M$ prior to conditioning on a readout record. First, when a system S and premeasurement probe E form equal-weight Schmidt-entangled branches, any branch permutation on S can be inverted by a compensating local unitary on E; QICM adopts Zurek's environment-assisted invariance (envariance) symmetry argument [105] as a structural motivation for equal prior propensities across equal-norm Schmidt branches (while recognizing that extending envariance to arbitrary unequal-weight superpositions requires additional continuity and noncontextuality assumptions, as analyzed by Schlosshauer and Fine [106], so that $P_k(0) = \int_{\Omega_k} \rho(q,0)\, d^d q$ remains an explicit probabilistic postulate of Postulate S; see also Quantum Darwinism [53]). Second, if a macroscopic experimental sampling distribution $f(q,t)$ initially departs from $\rho(q,t)$, conservative transport by their shared velocity field $\boldsymbol{v} = \nabla\Phi/m$ ($\partial_t f + \nabla\cdot(f\boldsymbol{v}) = 0$) preserves the fine-grained relative entropy and therefore requires coarse-grained mixing to drive relaxation.

For phase-space coarse-grained densities $(\overline{f}, \overline{\rho})$, assume an effective common advection–diffusion closure on a connected domain M:

$$\partial_t g + \nabla\cdot(\boldsymbol{v}_{\mathrm{c}} g) = \nabla\cdot(D_{\mathrm{mix}}\nabla g), \qquad g \in \{\overline{f}, \overline{\rho}\}.$$

Here $\boldsymbol{v}_{\mathrm{c}}$ is the common coarse-grained drift, D_{mix} is symmetric and positive definite, and both densities are normalized, strictly positive, and sufficiently regular for the following integrations by parts, with finite initial relative entropy. The domain has no boundary, or the densities decay sufficiently rapidly at infinity; on a bounded domain impose $\boldsymbol{v}_{\mathrm{c}}\cdot\boldsymbol{n} = 0$ and $\boldsymbol{n}\cdot D_{\mathrm{mix}}\nabla g = 0$. This diffusion closure is an additional model assumption about unresolved mixing, rather than a consequence of conservative transport alone. Under these conditions, the coarse-grained Kullback–Leibler relative entropy satisfies the nonincreasing quantum-equilibrium H-theorem [63]:

$$\begin{aligned} H_{\mathrm{KL}}[\overline{f}\|\overline{\rho}] &\equiv \int_M \overline{f}\ln\left(\frac{\overline{f}}{\overline{\rho}}\right) d^d q \geq 0, \\ \frac{dH_{\mathrm{KL}}[\overline{f}\|\overline{\rho}]}{dt} &= -\int_M \overline{f}\left|\nabla\ln\left(\frac{\overline{f}}{\overline{\rho}}\right)\right|^2_{D_{\mathrm{mix}}} d^d q \leq 0. \end{aligned} \tag{3.14}$$

At a given time, the entropy production vanishes if and only if $\overline{f} = \overline{\rho}$ almost everywhere on the connected domain: positive definiteness forces the logarithmic density ratio to be spatially constant, and normalization fixes that constant. With disconnected invariant components, the ratio can instead be constant separately on each component, so their masses must be specified before a unique equilibrium can be asserted.

Long-time convergence requires a further mixing estimate. Assume that, uniformly over the evolution, a constant $\lambda > 0$ exists such that

$$\int_M \overline{f}\left|\nabla\ln\left(\frac{\overline{f}}{\overline{\rho}}\right)\right|^2_{D_{\mathrm{mix}}} d^d q \geq 2\lambda H_{\mathrm{KL}}[\overline{f}\|\overline{\rho}].$$

Equation (3.14) then gives

$$\begin{aligned} H_{\mathrm{KL}}(t) &\leq e^{-2\lambda t} H_{\mathrm{KL}}(0), \\ \|\overline{f}(t) - \overline{\rho}(t)\|^2_{L^1(M)} &\leq 2H_{\mathrm{KL}}(t) \longrightarrow 0. \end{aligned}$$

Thus the assumed entropy-coercivity estimate yields convergence in total variation. If $\overline{\rho}$ is stationary under the closure, it is the unique normalized equilibrium in this class. Positivity of D_{mix} alone does not supply this quantitative estimate or pointwise convergence.

Entropy balance and equality condition. Set $r = \overline{f}/\overline{\rho}$. Conservation of the normalization of $\overline{f}$ gives

$$\frac{dH_{\mathrm{KL}}}{dt} = \int_M (\partial_t \overline{f}) \ln r \, d^d q - \int_M r \, \partial_t \overline{\rho} \, d^d q.$$

The common drift contributes

$$\int_M \boldsymbol{v}_{\mathrm{c}} \overline{f} \cdot \nabla \ln r \, d^d q - \int_M \boldsymbol{v}_{\mathrm{c}} \overline{\rho} \cdot \nabla r \, d^d q = 0.$$

For the diffusion terms, integration by parts and $\nabla \overline{f} = \overline{\rho} \nabla r + r \nabla \overline{\rho}$ yield

$$\begin{aligned} \frac{dH_{\mathrm{KL}}}{dt} &= -\int_M (\nabla \ln r)^{\mathsf{T}} D_{\mathrm{mix}} \nabla \overline{f} \, d^d q + \int_M (\nabla r)^{\mathsf{T}} D_{\mathrm{mix}} \nabla \overline{\rho} \, d^d q \\ &= -\int_M \frac{\overline{\rho}}{r} (\nabla r)^{\mathsf{T}} D_{\mathrm{mix}} \nabla r \, d^d q = -\int_M \overline{f} |\nabla \ln r|^2_{D_{\mathrm{mix}}} d^d q. \end{aligned}$$

This proves Eq. (3.14). Zero dissipation implies $\nabla \ln r = 0$ almost everywhere. Connectedness and normalization then give $r = 1$. The separate coercivity assumption gives $dH_{\mathrm{KL}}/dt \le -2\lambda H_{\mathrm{KL}}$; integration and the relative-entropy bound on total variation give the stated convergence. □

3.6 Topological dynamics during measurement: circulation conservation and nodal-pair annihilation

Proposition 3.2 (Circulation conservation and annihilation of oppositely oriented nodal pairs in open measurements)**.**

1. **Circulation conservation on loops that avoid the nodal set.** For a pure configuration measurement, $d\Phi_{\mathrm{back}} = 0$, evolution according to Eq. (3.8b) contains only a single-valued variational-gradient term wherever the fields remain regular. For a fixed closed loop C that remains in U_t throughout the time interval, integration gives conservation of its line-bundle holonomy, or vortex winding number n_l. A Poincaré–Hopf index sum is also conserved when the relevant topology is represented by a smooth vector field with isolated zeros on a fixed compact domain, with boundary data satisfying the hypotheses of the index theorem and no zero crossing the boundary. Under these respective conditions,

$$\begin{aligned} d\left(\oint_C \nabla \Phi(q,t) \cdot d\boldsymbol{l}\right) &= \oint_C \nabla \left(-\frac{\delta H}{\delta \rho}\right) \cdot d\boldsymbol{l} \, dt \equiv 0, \\ \sum_\alpha \mathrm{ind}(q_{0,\alpha}(t)) &= \mathrm{const.} \end{aligned} \tag{3.15}$$

   The second line of Eq. (3.15) concerns isolated zeros of such a vector field; it does not assign a Poincaré–Hopf index to an arbitrary nodal hypersurface. Circulation can change when a singularity crosses the chosen loop, outside the regularity conditions of the first line.

2. **Conditional local mechanism for annihilation of an oppositely oriented nodal pair.** As a multibranch superposition collapses onto a single eigenvalue support, the weights of the losing branches approach zero, $P_k(t) \to 0$. In regions where coherent branches overlap and admit a smooth signed real amplitude $u(q,t) = \pm\sqrt{\rho}$, evolving branch weights displace the interference zeros: in one dimension, a simple zero of u separates regions of opposite sign and persists locally while $\partial_q u \neq 0$, whereas two adjacent simple zeros of opposite slope—forming

an *oppositely oriented nodal pair*, distinct from a standing-wave antinode (nonzero amplitude extremum)—can coalesce at an interior double zero and annihilate.

For a generic interior annihilation at (q_*, t_*), the local smooth expansion has the form

$$u(q,t) = a(t-t_*) + b(q-q_*)^2 + o(|t-t_*| + |q-q_*|^2), \qquad ab > 0.$$

For $t < t_*$ sufficiently close to t_*, there are two simple zeros near $q_* \pm \sqrt{-(a/b)(t-t_*)}$ with opposite slopes; for $t > t_*$, these two local zeros are absent, changing the one-dimensional node count by $\Delta N_{\text{nodes}} = -2$. As a concrete solvable example, consider a coherent superposition of the oscillator ground state $u_0(q)$ and second excited state $u_2(q) \propto (2q^2/\Delta_{q,0}^2 - 1)u_0(q)$ with decaying excited-branch amplitude $c_2(t)$: its signed amplitude $u(q,t) \propto [1 - c_2(t) + (2c_2(t)/\Delta_{q,0}^2)q^2]u_0(q)$ has two symmetric simple nodes at $q_\pm(t) = \pm\Delta_{q,0}\sqrt{(c_2(t)-1)/(2c_2(t))}$ while $c_2(t) > 1$, which coalesce at $q_* = 0$ and annihilate as measurement reweighting drives $c_2(t)$ below 1. Global nodal evolution additionally depends on boundary crossings, possible pair creation, and Hamiltonian transport. Under strict branch-flow isolation with fixed disjoint supports, H2(a), reweighting the branches alone does not establish the overlapping-amplitude collision mechanism just described.

The global energy associated with a nodal rearrangement is governed by Eq. (3.9). A decrease of a local Fisher kinetic-energy contribution must be considered together with energy transport across the chosen region and changes in the flow, potential, and channel contributions. The sign of the local energy change is not fixed by the nodal count alone.

4 Kähler geometry of Ontic Phase Space and reconstruction of operator algebra

Standard quantum mechanics introduces two primitive postulates: physical states are vectors in complex Hilbert space, and observables are self-adjoint linear operators on that space. Starting from Axioms I and II, this section addresses why a theory whose objective ontology consists of real statistical fields $(\rho, [\Phi])$ acquires the algebra of complex Hilbert space and Heisenberg commutators.

Remark (Physical intuition and interpretation: symplectic volume, information distance, and i). Real Ontic Phase Space carries two natural geometric structures: the canonical symplectic form Ω [Eq. (4.1)], which pairs $\delta\rho$ with $\delta\Phi$ to measure reversible action area under Liouville volume conservation, and the rescaled Fisher–Rao metric G_{FR}, which measures statistical distinguishability between neighboring field states. As summarized in Fig. 2, Theorem 1 unifies these structures into an infinite-dimensional Kähler geometry via a quarter-turn tangent rotation J ($J^2 = -I$) satisfying $\Omega(u,v) = G_{\text{FR}}(Ju, v)$, so that the complex coordinate map $\psi = \sqrt{\rho}e^{i\Phi/\hbar}$ represents J as multiplication by i and combines $G_{\text{FR}} + i\Omega$ into the Hilbert-space Hermitian inner product. Physically, just as an electromagnetic wave requires both electric and magnetic fields rotated by 90° in phase to propagate while conserving energy, reversible probability transport on $\mathcal{M}_{\text{ontic}}$ requires coupling density variations $\delta\rho$ and phase variations $\delta\Phi$ through the quarter-turn operator J: unitary Hilbert-space evolution is simply the unique real-field flow that simultaneously conserves phase-space action volume (Ω) and statistical distinguishability (G_{FR}). Theorem 2 then shows that generators preserving both geometric structures correspond one-to-one to self-adjoint operators whose commutators mirror the real-field symplectic Poisson bracket.

4.1 Theorem 1: Kähler correspondence and complex Hilbert-space equivalence

Theorem 1 (Kähler correspondence of Ontic Phase Space and Hilbert-space equivalence)**.** Equip $\mathcal{M}_{\text{ontic}} = T^*\mathcal{P}_{\text{phys}}(M)/h\mathbb{Z}$ with the canonical symplectic form Ω of Axiom I and the Sasaki–Fisher–

Schematic Overview: Madelung–Kähler Isometric Symplectomorphism $\mathcal{M}_K$

Real Ontic Phase Space $\mathcal{M}_{\text{ontic}} = \{(\rho, [\Phi])\}$	**Bijective Map**	**Complex Hilbert Sphere** $S(\mathcal{H}) \subset L^2(M, \mathbb{C})$
1. **Canonical Symplectic 2-Form:** $\Omega = \int_M \delta\rho \wedge \delta\Phi \, d^d q$ (reversible action area; Liouville volume conservation)	$\xrightarrow{\psi = \sqrt{\rho}\, e^{i\Phi/\hbar}}$	1. **Hermitian Inner Product:** $2\hbar\langle\delta\psi_1|\delta\psi_2\rangle = G_{\text{FR}} + i\Omega$ (real part: Fisher metric; imaginary part: symplectic form)
2. **Rescaled Fisher–Rao Metric:** $G_{\text{FR}} = \int_M \left[\frac{\hbar}{2\rho}\delta\rho^2 + \frac{2\rho}{\hbar}\delta\Phi^2\right] d^d q$ (statistical distinguishability distance)	$\xleftarrow[\rho=\lvert\psi\rvert^2,\ \Phi=\hbar\arg\psi]{d\mathcal{M}_K \circ J = i\, d\mathcal{M}_K}$	2. **Kähler Isometric Generators:** $A[\rho, \Phi] = \langle\psi|\hat{A}|\psi\rangle \iff \hat{A} = \hat{A}^\dagger$ (simultaneous preservation of G_{FR} and Ω)
3. **Complex Structure Rotation J (on $\mathcal{T}^{\text{red}}\mathcal{M}_{\text{ontic}}$):** $J(\delta\rho, \delta\Phi) = \left(-\frac{2\rho}{\hbar}\delta\Phi, \frac{\hbar}{2\rho}\delta\rho\right)$, $\int \delta\rho = 0$, $\int \rho\, \delta\Phi = 0$, $J^2 = -I$, $\Omega(u, v) = G_{\text{FR}}(Ju, v)$	$\iff$	3. **Lie-Algebra Isomorphism:** $\{A, B\}_\Omega = \frac{1}{i\hbar}\langle\psi|[\hat{A}, \hat{B}]|\psi\rangle$ (symplectic Poisson bracket $\leftrightarrow$ commutator)

Figure 2: Ontic Kähler geometry and complex Hilbert-space correspondence. On the real Ontic Phase Space $\mathcal{M}_{\text{ontic}}$, the canonical symplectic form Ω measures reversible action area, the rescaled Fisher–Rao metric G_{FR} measures statistical distinguishability, and the tangent rotation J ($J^2 = -I$) locks them into a compatible Kähler triple. The globally single-valued Madelung–Kähler map $\mathcal{M}_K$ converts J into multiplication by i and induces the Hermitian inner product, self-adjoint observables, and Heisenberg commutation relations (Theorems 1–2).

Rao cotangent metric G_{FR} [73, 78] calibrated by $\hbar = 2hj_0$ (where the density block is the rescaled Fisher–Rao metric and the phase block is its dual kinetic cotangent lift):

$$\begin{aligned} \Omega\big((\delta\rho_1, \delta\Phi_1), (\delta\rho_2, \delta\Phi_2)\big) &= \int_M (\delta\rho_1\delta\Phi_2 - \delta\Phi_1\delta\rho_2)\, d^d q, \\ G_{\text{FR}}\big((\delta\rho_1, \delta\Phi_1), (\delta\rho_2, \delta\Phi_2)\big) &= \int_M \left(\frac{\hbar}{2\rho}\delta\rho_1\delta\rho_2 + \frac{2\rho}{\hbar}\delta\Phi_1\delta\Phi_2\right) d^d q. \end{aligned} \tag{4.1}$$

Because probability normalization imposes $\int_M \delta\rho\, d^d q = 0$, a spatially constant phase shift $u_0 = (0, c)$ satisfies $\Omega(u_0, v) = -c\int_M \delta\rho_v\, d^d q = 0$ and generates the global $U(1)$ gauge fiber $\Phi \mapsto \Phi + c$. Define the reduced horizontal tangent space orthogonal to global phase shifts by

$$\mathcal{T}^{\text{red}}_{(\rho,[\Phi])}\mathcal{M}_{\text{ontic}} \equiv \left\{(\delta\rho, \delta\Phi) \;\middle|\; \int_M \delta\rho(q)\, d^d q = 0,\ \int_M \rho(q)\, \delta\Phi(q)\, d^d q = 0\right\},$$

and define the tangent operator J and the Madelung–Kähler coordinate map M_K by

$$\begin{aligned} J(\delta\rho, \delta\Phi) &\equiv \left(-\frac{2\rho}{\hbar}\delta\Phi, \frac{\hbar}{2\rho}\delta\rho\right), \\ M_K : (\rho(q,t), [\Phi(q,t)]) &\longmapsto \psi(q,t) \equiv \sqrt{\rho(q,t)}\exp\left(\frac{i}{\hbar}\Phi(q,t)\right). \end{aligned} \tag{4.2}$$

Then: (1) J maps $\mathcal{T}^{\text{red}}_{(\rho,[\Phi])}\mathcal{M}_{\text{ontic}}$ onto itself, and $(G_{\text{FR}}, \Omega, J)$ is a nondegenerate, compatible infinite-dimensional Kähler structure on the projective state manifold $\mathcal{M}_{\text{ontic}}/U(1) \cong \mathbb{P}(\mathcal{H})$ [73, 75–78], satisfying $J^2 = -I$, $\Omega(u, v) = G_{\text{FR}}(Ju, v)$, and $G_{\text{FR}}(Ju, Jv) = G_{\text{FR}}(u, v)$; (2) under the holonomy rule (3.1), half-period nodal jump (3.2), and technical condition H1, M_K lifts in the bosonic/trivial sector ($\chi = 0$) to a globally single-valued wave function on the unit sphere $S(\mathcal{H}_{\text{phys}}) = \{\psi \in H^2_{\text{loc}}(M, \mathbb{C}) \cap H^1(M, \mathbb{C}) \mid \int_M |\psi|^2 d^d q = 1\} \subset \mathcal{H} = L^2(M, \mathbb{C})$ (and in a nontrivial $\mathbb{Z}_2$ torsion sector $\chi = 1/2$ to a single-valued section of the corresponding flat spinor line bundle over M), mapping

$\mathcal{T}^{\mathrm{red}}_{(\rho,[\Phi])}\mathcal{M}_{\mathrm{ontic}}$ isometrically and symplectically onto the horizontal tangent space $\mathcal{T}^{\mathrm{hor}}_{\psi}S(\mathcal{H}) = \{\delta\psi \in H^1(M,\mathbb{C}) \mid \langle\psi|\delta\psi\rangle = 0\}$ according to the exact Hermitian identity

$$2\hbar\int_M \delta\psi_1^*(q)\delta\psi_2(q)\,d^dq = G_{\mathrm{FR}}\big((\delta\rho_1,\delta\Phi_1),(\delta\rho_2,\delta\Phi_2)\big) + i\Omega\big((\delta\rho_1,\delta\Phi_1),(\delta\rho_2,\delta\Phi_2)\big). \tag{4.3}$$

Remark (Physical intuition and interpretation: the Hermitian product and PBR). Equation (4.3) resolves the abstract Hermitian inner product into two concrete physical quantities [75–77, 95]: its real part G_{FR} quantifies statistical distinguishability under experimental sampling, while its imaginary part Ω measures the geometric action phase accumulated around a closed phase-space loop. Kähler compatibility renders information-distance preservation and symplectic volume preservation two facets of a single unitary symmetry. Because the Madelung–Kähler map defines a bijection between Ontic Phase Space and projective Hilbert space, distinct pure states correspond to disjoint ontic field configurations $(\rho_1,[\Phi_1]) \neq (\rho_2,[\Phi_2])$ with zero epistemic overlap—establishing psi-onticity in the Harrigan–Spekkens sense [25, 26] directly from real phase-space geometry (complementing the Pusey–Barrett–Rudolph exclusion of epistemic overlap under preparation independence [27]).

Proof. **Step 1: complex structure and Kähler compatibility.** For $u = (\delta\rho_1,\delta\Phi_1)$ and $v = (\delta\rho_2,\delta\Phi_2)$, apply J twice:

$$\begin{aligned} J^2(\delta\rho_1,\delta\Phi_1) &= J\left(-\frac{2\rho}{\hbar}\delta\Phi_1,\frac{\hbar}{2\rho}\delta\rho_1\right)\\ &= \left(-\frac{2\rho}{\hbar}\left(\frac{\hbar}{2\rho}\delta\rho_1\right),\frac{\hbar}{2\rho}\left(-\frac{2\rho}{\hbar}\delta\Phi_1\right)\right)\\ &= (-\delta\rho_1,-\delta\Phi_1) = -I(\delta\rho_1,\delta\Phi_1). \end{aligned} \tag{4.4}$$

Substitution of Ju into the metric gives

$$\begin{aligned} G_{\mathrm{FR}}(Ju,v) &= \int_M\left[\frac{\hbar}{2\rho}\left(-\frac{2\rho}{\hbar}\delta\Phi_1\right)\delta\rho_2 + \frac{2\rho}{\hbar}\left(\frac{\hbar}{2\rho}\delta\rho_1\right)\delta\Phi_2\right]d^dq\\ &= \int_M(\delta\rho_1\delta\Phi_2 - \delta\Phi_1\delta\rho_2)d^dq = \Omega(u,v). \end{aligned} \tag{4.5}$$

Also,

$$G_{\mathrm{FR}}(Ju,Jv) = \Omega(u,Jv) = -\Omega(Jv,u) = -G_{\mathrm{FR}}(J^2v,u) = G_{\mathrm{FR}}(v,u) = G_{\mathrm{FR}}(u,v).$$

Moreover, for any $u = (\delta\rho,\delta\Phi) \in \mathcal{T}^{\mathrm{red}}_{(\rho,[\Phi])}\mathcal{M}_{\mathrm{ontic}}$, the image $Ju = (-(2\rho/\hbar)\delta\Phi,(\hbar/2\rho)\delta\rho)$ satisfies $\int_M(-(2\rho/\hbar)\delta\Phi)\,d^dq = -(2/\hbar)\int_M\rho\,\delta\Phi\,d^dq = 0$ and $\int_M\rho((\hbar/2\rho)\delta\rho)\,d^dq = (\hbar/2)\int_M\delta\rho\,d^dq = 0$, confirming that J preserves $\mathcal{T}^{\mathrm{red}}_{(\rho,[\Phi])}\mathcal{M}_{\mathrm{ontic}}$ and removes the global-phase kernel $u_0 = (0,c)$. Under the tangent map (4.6), the two constraints $\int_M\delta\rho\,d^dq = 0$ and $\int_M\rho\,\delta\Phi\,d^dq = 0$ are equivalent to $\mathrm{Re}\langle\psi|\delta\psi\rangle = 0$ and $\mathrm{Im}\langle\psi|\delta\psi\rangle = 0$, i.e., $\langle\psi|\delta\psi\rangle = 0$, which defines the Fubini–Study horizontal subspace of projective Hilbert space $\mathbb{P}(\mathcal{H})$ [75, 76]. Since $\Omega = -\delta(\int_M\Phi\,\delta\rho\,d^dq)$ is exact and closed ($\delta\Omega = 0$), $(G_{\mathrm{FR}},\Omega,J)$ forms a nondegenerate compatible Kähler structure on the reduced state space.

Step 2: tangent map and Hermitian product. Axiom I makes $[\Phi]$ periodic with $h = 2\pi\hbar$, so $e^{i(\Phi+nh)/\hbar} = e^{i\Phi/\hbar}e^{2\pi in} = e^{i\Phi/\hbar}$. Across a first-order nodal surface, a jump $\pi\hbar$ gives $e^{i\pi} = -1$ and restores the nonnegative root $\sqrt{\rho} = |u|$ to the smooth signed amplitude $u = \pm\sqrt{\rho}$. Thus ψ is globally single-valued and smooth. Its first Fréchet variation is

$$\delta\psi_k(q) = \left(\frac{\delta\rho_k(q)}{2\sqrt{\rho(q)}} + i\frac{\sqrt{\rho(q)}}{\hbar}\delta\Phi_k(q)\right)\exp\left(\frac{i}{\hbar}\Phi(q)\right), \qquad k = 1,2. \tag{4.6}$$

Applying J before the tangent map yields

$$dM_K(J(\delta\rho_k, \delta\Phi_k)) = \left[-\frac{\delta\Phi_k\sqrt{\rho}}{\hbar} + i\frac{\delta\rho_k}{2\sqrt{\rho}}\right] e^{i\Phi/\hbar} = i\delta\psi_k.$$

The real operator J thus becomes multiplication by i. Multiplying the integrand $\delta\psi_1^*\delta\psi_2$ (which has dimension L^{-d}) by $2\hbar$ (dimension ML^2T^{-1}) and expanding term by term gives

$$\begin{aligned} &2\hbar\int_M \left(\frac{\delta\rho_1}{2\sqrt{\rho}} - i\frac{\sqrt{\rho}}{\hbar}\delta\Phi_1\right)\left(\frac{\delta\rho_2}{2\sqrt{\rho}} + i\frac{\sqrt{\rho}}{\hbar}\delta\Phi_2\right) d^dq \\ &= 2\hbar\int_M \left[\left(\frac{\delta\rho_1\delta\rho_2}{4\rho} + \frac{\rho}{\hbar^2}\delta\Phi_1\delta\Phi_2\right) + \frac{i}{2\hbar}(\delta\rho_1\delta\Phi_2 - \delta\Phi_1\delta\rho_2)\right] d^dq \\ &= \int_M \left(\frac{\hbar}{2\rho}\delta\rho_1\delta\rho_2 + \frac{2\rho}{\hbar}\delta\Phi_1\delta\Phi_2\right) d^dq + i\int_M (\delta\rho_1\delta\Phi_2 - \delta\Phi_1\delta\rho_2) d^dq. \end{aligned} \tag{4.7}$$

Both sides have the dimension of action ML^2T^{-1}, establishing Eq. (4.3). □

4.2 Theorem 2: observables, Poisson–commutator correspondence, and premeasurement

Theorem 2 (Self-adjoint observables, symplectic brackets, and premeasurement)**.**

1. **Kähler isometric generators and self-adjoint operators.** Every strongly continuous one-parameter family of Kähler automorphisms connected to the identity that preserves both Ω and G_{FR} is generated by a real functional $A[\rho, \Phi]$ which, in the coordinate $\psi = \sqrt{\rho}e^{i\Phi/\hbar}$ on its dense operator domain $\mathcal{D}(\hat{A}) \subset \mathcal{H}$, has a unique quadratic expectation representation $A[\rho, \Phi] = \langle\psi|\hat{A}|\psi\rangle = \int_M \psi^*(q)(\hat{A}\psi)(q)d^dq$ with $\hat{A} = \hat{A}^\dagger$ [75, 76, 78].
2. **Real-field Poisson brackets and Heisenberg commutators.** For $A = \langle\psi|\hat{A}|\psi\rangle$ and $B = \langle\psi|\hat{B}|\psi\rangle$ evaluated on a common invariant dense domain $\mathcal{D} \subset H^2_{\mathrm{loc}}(M, \mathbb{C}) \cap H^1(M, \mathbb{C})$,
$$\begin{aligned} \{A, B\}_\Omega &\equiv \int_M \left(\frac{\delta A}{\delta\rho(q)}\frac{\delta B}{\delta\Phi(q)} - \frac{\delta A}{\delta\Phi(q)}\frac{\delta B}{\delta\rho(q)}\right) d^dq \\ &= \frac{1}{i\hbar}\langle\psi|[\hat{A}, \hat{B}]|\psi\rangle. \end{aligned} \tag{4.8}$$
3. **Unitary premeasurement of nonconfiguration observables.** For a discrete-spectrum or binned self-adjoint observable $\hat{A} = \sum_k a_k\hat{P}_k$ (such as momentum, energy, spin projection, or displaced parity $\Pi(\alpha)$), a bilinear system–pointer coupling $\hat{H}_{\mathrm{pre}} = g\hat{A} \otimes \hat{p}_{\mathrm{det}}$ generates a symplectic and metric-preserving Kähler flow $\hat{U}_{\mathrm{pre}} = \exp(-i\tau_{\mathrm{pre}}\hat{H}_{\mathrm{pre}}/\hbar)$ (e.g., Stern–Gerlach gradients or dispersive beam-splitter couplings). For an initial pointer packet of compact support (with diameter smaller than the branch shift $g\tau_{\mathrm{pre}} \min_{j\neq k} |a_j - a_k|$), $\hat{U}_{\mathrm{pre}}$ maps orthogonal eigenspaces onto strictly disjoint detector supports Ω_k, preserving each initial Born weight exactly: $\langle\psi(0)|\hat{P}_k|\psi(0)\rangle = \int_{\Omega_k} \rho_{\mathrm{pre}}(q)d^dq$ (for a Gaussian pointer of width σ_{det}, the supports Ω_k partition pointer space into Voronoi decision cells with exponentially suppressed tail overlap error $\varepsilon_{\mathrm{ov}} = \frac{1}{2}\,\mathrm{erfc}[g\tau_{\mathrm{pre}}\Delta a_{\mathrm{min}}/(2\sqrt{2}\sigma_{\mathrm{det}})]$).

Remark (Physical intuition and interpretation: noncommutativity and spatial readout). Classical position and momentum are treated as passive numerical readings. In the real-field picture, $Q[\rho] = \int q\rho\, d^dq$ depends only on density, and its symplectic flow tilts the conjugate potential, $\delta\Phi \propto q$, changing $\nabla\Phi/m$. Conversely, $P[\rho, \Phi] = \int \rho\nabla\Phi\, d^dq$ depends on the potential gradient and generates density translation, $\delta\rho \propto -\nabla\rho$. Reversing the order of these transformations encloses a symplectic loop of area $\hbar$. Thus $[\hat{q}, \hat{p}] = i\hbar$ expresses the phase-space symplectic geometry of the conjugate field pair, Eq. (4.8).

Part 3 clarifies the experimental role of premeasurement. A probe reads momentum or spin by separating spatial channels: Stern–Gerlach gradients deflect atomic beams, and gratings or time of flight resolve momentum. A symplectic, information-distance-preserving $\hat{U}_{\mathrm{pre}}$ maps eigenbranches into distinguishable supports Ω_k; Axiom III then supplies irreversible configuration-space readout. The same account addresses non-fragmentation in the Heptalemma [33] and Kochen–Specker or sheaf-theoretic contextuality [61, 62, 107, 108]. Incompatible settings a, a' lack a single static joint probability distribution because measurement actively applies different noncommuting premeasurement rotations $\hat{U}_{\mathrm{pre}}(a)$ and $\hat{U}_{\mathrm{pre}}(a')$ to the same globally consistent $\mathcal{M}_{\mathrm{ontic}}$, rather than passively reading predetermined point values or fragmenting the objective world.

Proof. **Step 1: Wigner–Kähler isometries and self-adjoint generators.** By Eq. (4.3), a one-parameter transformation preserves G_{FR} and Ω exactly when its image under M_K preserves the Hermitian product. These transformations form the complex-linear unitary group $U(H)$. By Stone's theorem [109], the infinitesimal generator of a strongly continuous group $\hat{U}(s) = \exp(-is\hat{A}/\hbar)$ is self-adjoint. Its Hamiltonian flow obeys $i\hbar\partial_s\psi = \hat{A}\psi$. Integration with Ω determines $A = \langle\psi|\hat{A}|\psi\rangle$ up to an irrelevant constant.

Step 2: functional chain rule and Eq. (4.8). We use $\iota_{X_F}\Omega = dF$, so that $X_F = (\delta F/\delta\Phi, -\delta F/\delta\rho)$ and $dA/dt = \{A, H\}_\Omega$ along the Hamiltonian flow of H. This convention agrees with the closed part of Axiom III. Using $\psi = \sqrt{\rho}e^{i\Phi/\hbar}$, $\psi^* = \sqrt{\rho}e^{-i\Phi/\hbar}$, and self-adjointness, the functional derivatives are

$$\begin{aligned}\frac{\delta A}{\delta\rho(q)} &= \frac{1}{2\sqrt{\rho(q)}}\left[e^{-i\Phi/\hbar}(\hat{A}\psi) + e^{i\Phi/\hbar}(\hat{A}\psi)^*\right],\\ \frac{\delta A}{\delta\Phi(q)} &= \frac{i\sqrt{\rho(q)}}{\hbar}\left[-e^{-i\Phi/\hbar}(\hat{A}\psi) + e^{i\Phi/\hbar}(\hat{A}\psi)^*\right].\end{aligned} \tag{4.9}$$

By self-adjointness $\hat{A} = \hat{A}^\dagger$, integrating the second equation in (4.9) gives $\int_M(\delta A/\delta\Phi)\,d^dq = 0$, while integrating the first gives $\int_M \rho(\delta A/\delta\rho)\,d^dq = \langle\psi|\hat{A}|\psi\rangle = A$. Projecting the Hamiltonian vector field $X_A = (\delta A/\delta\Phi, -\delta A/\delta\rho)$ onto the reduced horizontal tangent space $\mathcal{T}^{\mathrm{red}}_{(\rho,[\Phi])}\mathcal{M}_{\mathrm{ontic}}$ of Theorem 1 replaces $\hat{A}$ by the centered operator $\hat{A}_0 \equiv \hat{A} - \langle\hat{A}\rangle\hat{I}$, subtracting the spatial constant $\langle\hat{A}\rangle$ from $\delta A/\delta\rho(q)$. Because $\int_M(\delta B/\delta\Phi)\,d^dq = 0$, this constant shift lies in the kernel of Ω and leaves the Poisson bracket $\{A, B\}_\Omega$ invariant. The corresponding derivatives of B have the same form, with coefficient product $[1/(2\sqrt{\rho})][i\sqrt{\rho}/\hbar] = i/(2\hbar)$. The same-phase terms $e^{-2i\Phi/\hbar}(\hat{A}\psi)(\hat{B}\psi)$ and $e^{2i\Phi/\hbar}(\hat{A}\psi)^*(\hat{B}\psi)^*$ cancel, leaving

$$\frac{\delta A}{\delta\rho}\frac{\delta B}{\delta\Phi} - \frac{\delta A}{\delta\Phi}\frac{\delta B}{\delta\rho} = \frac{1}{i\hbar}\left[(\hat{A}\psi)^*(\hat{B}\psi) - (\hat{B}\psi)^*(\hat{A}\psi)\right]. \tag{4.10}$$

The surviving coefficient is $-i/\hbar = 1/(i\hbar)$. Integration on a common invariant operator domain, together with $\hat{A}^\dagger = \hat{A}$ and $\hat{B}^\dagger = \hat{B}$, gives

$$\{A, B\}_\Omega = \frac{1}{i\hbar}\int_M(\psi^*\hat{A}\hat{B}\psi - \psi^*\hat{B}\hat{A}\psi)d^dq = \frac{1}{i\hbar}\langle\psi|[\hat{A}, \hat{B}]|\psi\rangle.$$

As a direct sign check in one dimension, $Q = \int q\rho\,dq$ and $P = \int\rho\,\partial_q\Phi\,dq$ give $\{Q, P\}_\Omega = -\int q\,\partial_q\rho\,dq = 1$ under H1, consistently with $[\hat{q}, \hat{p}] = i\hbar$. Finally, for $\hat{A} = \sum_k a_k\hat{P}_k$, choose the bilinear von Neumann generator $\hat{H}_{\mathrm{pre}} = g\hat{A}\otimes\hat{p}_{\mathrm{det}}$. Its Kähler isometric flow $\hat{U}_{\mathrm{pre}} = \exp(-i\tau_{\mathrm{pre}}\hat{H}_{\mathrm{pre}}/\hbar)$ translates the eigenbranches into supports Ω_k. By Eq. (4.3), this flow preserves $\langle\psi(0)|\hat{P}_k|\psi(0)\rangle = \int_{\Omega_k}\rho_{\mathrm{pre}}(q)d^dq$. □

5 Core nonrelativistic theorems, physical interpretation, and mathematical proofs

This section derives and proves the five core theorems of nonrelativistic quantum mechanics, Theorems 3–7, from Axioms I–III and the technical conditions. Before each proof, an account of the physical mechanism connects the mathematics to the information-capacity picture of uncertainty, minimum resolution time, the quantum potential, quantized stationary spectra, and measurement collapse.

5.1 Theorem 3: momentum-variance decomposition, uncertainty relations, and positive phase-space entropy

Theorem 3 (Momentum-variance decomposition, uncertainty relations, and positive phase-space entropy)**.** The following statements hold for every one-dimensional physical state $(\rho(q,t),[\Phi(q,t)])$ satisfying Axioms I–III and technical condition H1, and for its general d-dimensional counterpart.

1. The total momentum variance Δ_p^2 decomposes orthogonally into the variance of the macroscopic flow fluctuations and the local Fisher-information variance:
$$\Delta_p^2 = \int_{-\infty}^{\infty} \rho(q,t)\big(\partial_q\Phi(q,t) - \bar{p}(t)\big)^2\, dq + (hj_0)^2 I_F[\rho]. \tag{5.1}$$
2. The determinant of the phase-space covariance matrix Σ satisfies the Robertson–Schrödinger uncertainty relation [110, 111] and the Heisenberg uncertainty relation [15]:
$$\det\Sigma = \Delta_q^2\Delta_p^2 - \sigma_{qp}^2 \geq (hj_0)^2 = \frac{\hbar^2}{4} \quad\Longrightarrow\quad \Delta_q\Delta_p \geq \frac{\hbar}{2}. \tag{5.2}$$
Equality $\Delta_q\Delta_p = \hbar/2$ holds if and only if $\rho(q,t)$ is Gaussian and the information-flow momentum $\partial_q\Phi(q,t) = \bar{p}(t)$ is spatially constant.
3. The Gaussian-reference covariance differential entropy $S_{\mathrm{state}}[\Sigma] \equiv \frac{1}{2}\ln[((2\pi e)^2\det\Sigma)/h^2]$, defined relative to the Planck-cell measure $dq\,dp/h$, is bounded below for every physical state satisfying H1 by
$$S_{\mathrm{state}}[\Sigma] \geq 1 - \ln 2 \approx 0.3069 \text{ nat} > 0. \tag{5.3}$$

Remark (Physical intuition and interpretation: why stronger spatial localization entails larger momentum fluctuations). Standard quantum-mechanics texts usually attribute the Heisenberg uncertainty relation to the noncommutativity of the position and momentum operators, $\hat{q}$ and $\hat{p}$, or to the Fourier bandwidth of a wave packet. Theorem 3 identifies a more direct physical mechanism. Equation (5.1) expresses the total momentum fluctuations as the sum of two orthogonal contributions. The first is the spatially nonuniform fluctuation of the macroscopic fluid velocity, $\int \rho(\partial_q\Phi - \bar{p})^2\, dq$. In a completely stationary state where $\partial_q\Phi = 0$, this contribution vanishes. The system nevertheless retains the irreducible intrinsic osmotic momentum variance $(hj_0)^2 I_F[\rho]$. To compare localization scales without changing the shape, take a normalized profile r with finite Fisher information and set
$$\rho_\ell(q) = \ell^{-d} r(x), \qquad x = (q-\bar{q})/\ell,$$
$$\nabla_q \ln\rho_\ell(q) = \ell^{-1}\nabla_x \ln r(x), \qquad I_F[\rho_\ell] = \ell^{-2} I_F[r].$$
Thus the logarithmic gradient grows as ℓ^{-1} at a fixed scaled coordinate, and its root-mean-square magnitude has the same scaling; the Fisher information grows as ℓ^{-2}. In one dimension, Δ_q scales linearly with ℓ. For the Gaussian (5.10), $\partial_q \ln\rho = -(q-\bar{q})/\Delta_q^2$ instead scales as Δ_q^{-2} at a fixed physical displacement $q-\bar{q}$; its root-mean-square magnitude is $1/\Delta_q$. These are different

comparisons, consistent with the Stam–Cramér–Rao information inequality [104], $\Delta_q^2 I_F[\rho] \geq 1$. The Heisenberg relation is thus the direct manifestation of the statistical Cramér–Rao lower bound in a physical phase space with minimum angular-momentum scale $hj_0 = \hbar/2$.

Equation (5.3) is a direct logarithmic restatement of the Robertson–Schrödinger determinant bound $\det \Sigma \geq \hbar^2/4$ in Eq. (5.2): whereas the differential entropy of a classical phase-space distribution relative to $dq\,dp/h$ can approach $-\infty$ as its phase-space support shrinks to zero, the minimum symplectic area $\sqrt{\det \Sigma} \geq \hbar/2$ fixes the lower bound of $S_{\text{state}}[\Sigma]$ at $1 - \ln 2$ (where the numerical value $1 - \ln 2$ includes the Gaussian $(2\pi e)$ normalization offset, and $S_{\text{state}}[\Sigma]$ is distinct from both von Neumann entropy and thermodynamic Gibbs entropy).

Proof. **Step 1: derivation of the momentum-variance decomposition** (5.1) **from Axiom III.** Consider a one-dimensional system, $d = 1$. Subtracting the potential energy $\int \rho V\, dq$ from the total energy functional (3.6) of Axiom III gives the total kinetic-energy expectation

$$\begin{aligned}\langle T \rangle = \frac{1}{2m} \int_{-\infty}^{\infty} \rho(q,t) \left(\frac{\partial \Phi(q,t)}{\partial q} \right)^2 dq \\ + \frac{(hj_0)^2}{2m} \int_{-\infty}^{\infty} \rho(q,t) \left(\frac{\partial \ln \rho(q,t)}{\partial q} \right)^2 dq.\end{aligned} \tag{5.4}$$

By the definition

$$I_F[\rho] = \int_{-\infty}^{\infty} \rho (\partial_q \ln \rho)^2\, dq,$$

the second term in (5.4) is $[(hj_0)^2/(2m)] I_F[\rho]$. The constitutive relation for the second momentum moment in Axiom III agrees with the quadratic-form expectation of the self-adjoint momentum operator $\hat{p} = -i\hbar\partial_q$ in Theorem 2. It expresses the kinetic energy in terms of the mean momentum $\bar{p}$ and its variance:

$$\langle T \rangle \equiv \frac{\langle p^2 \rangle}{2m} = \frac{\bar{p}^2 + \Delta_p^2}{2m}. \tag{5.5}$$

Using $\bar{p}(t) \equiv \int_{-\infty}^{\infty} \rho(q,t) \partial_q \Phi(q,t)\, dq$ and $\int_{-\infty}^{\infty} \rho(q,t)\, dq = 1$, expand the flow-fluctuation integral:

$$\begin{aligned}\int_{-\infty}^{\infty} \rho (\partial_q \Phi - \bar{p})^2\, dq &= \int_{-\infty}^{\infty} \rho (\partial_q \Phi)^2\, dq - 2\bar{p} \int_{-\infty}^{\infty} \rho \partial_q \Phi\, dq + \bar{p}^2 \int_{-\infty}^{\infty} \rho\, dq \\ &= \int_{-\infty}^{\infty} \rho (\partial_q \Phi)^2\, dq - \bar{p}^2.\end{aligned} \tag{5.6}$$

Substituting $\int \rho (\partial_q \Phi)^2\, dq = \bar{p}^2 + \int \rho (\partial_q \Phi - \bar{p})^2\, dq$ into (5.4) and combining the result with (5.5) gives

$$\begin{aligned}\frac{\bar{p}^2 + \Delta_p^2}{2m} = \frac{\bar{p}^2}{2m} + \frac{1}{2m} \int_{-\infty}^{\infty} \rho(q,t) \left(\partial_q \Phi(q,t) - \bar{p}(t) \right)^2 dq \\ + \frac{(hj_0)^2}{2m} I_F[\rho].\end{aligned} \tag{5.7}$$

Multiplication by $2m$ and cancellation of $\bar{p}^2$ establish (5.1).

Step 2: first-principles proof of $\Delta_q^2 I_F[\rho] \geq 1$ and the necessary and sufficient Gaussian equality condition. Let $\rho(q,t)$ satisfy H1 and have finite second central moment $\Delta_q^2 = \int_{-\infty}^{\infty} (q - \bar{q})^2 \rho(q,t)\, dq < \infty$. Integration by parts yields

$$\begin{aligned}\int_{-\infty}^{\infty} (q - \bar{q}) \frac{\partial \rho(q,t)}{\partial q}\, dq = \left[(q - \bar{q}) \rho(q,t) \right]_{q=-\infty}^{q=+\infty} \\ - \int_{-\infty}^{\infty} \rho(q,t) \frac{\partial (q - \bar{q})}{\partial q}\, dq = -1.\end{aligned} \tag{5.8}$$

The chain rule $\partial\rho/\partial q = \rho(\partial \ln\rho/\partial q)$ identifies this integral as the inner product of the two square-integrable functions $f(q) \equiv \sqrt{\rho(q,t)}(q-\bar{q})$ and $g(q) \equiv \sqrt{\rho(q,t)}(\partial \ln\rho/\partial q)$. Squaring this inner product and applying the integral Cauchy–Schwarz inequality, $(\int fg\,dq)^2 \le (\int f^2\,dq)(\int g^2\,dq)$, gives

$$\begin{aligned} 1 &= \left(\int_{-\infty}^{\infty} f(q)g(q)\,dq\right)^2 \\ &\le \left(\int_{-\infty}^{\infty}(q-\bar{q})^2\rho\,dq\right)\left(\int_{-\infty}^{\infty}\rho\left(\frac{\partial\ln\rho}{\partial q}\right)^2 dq\right) = \Delta_q^2 I_F[\rho]. \end{aligned} \tag{5.9}$$

This proves $\Delta_q^2 I_F[\rho] \ge 1$. Equality in (5.9) holds if and only if g and f are linearly dependent: a real constant $c = -1/\Delta_q^2$, independent of q, must satisfy $\partial_q \ln\rho(q,t) = -(q-\bar{q})/\Delta_q^2$. Integration and normalization give the unique extremizing Gaussian density

$$\rho(q,t) = \frac{1}{\sqrt{2\pi}\,\Delta_q(t)}\exp\left[-\frac{(q-\bar{q}(t))^2}{2\Delta_q^2(t)}\right]. \tag{5.10}$$

Step 3: proof of the Robertson–Schrödinger inequality (5.2)**, positive phase-space entropy** (5.3)**, and the multimode Williamson extension.** Multiply (5.1) by Δ_q^2 and subtract the square of the symmetric covariance, $\sigma_{qp}^2 = [\int \rho(q-\bar{q})(\partial_q\Phi - \bar{p})\,dq]^2$:

$$\begin{aligned} \det\Sigma &= \Delta_q^2\Delta_p^2 - \sigma_{qp}^2 \\ &= \left[\left(\int\rho(q-\bar{q})^2\,dq\right)\left(\int\rho(\partial_q\Phi-\bar{p})^2\,dq\right)\right. \\ &\quad \left. - \left(\int\rho(q-\bar{q})(\partial_q\Phi-\bar{p})\,dq\right)^2\right] \\ &\quad + (hj_0)^2\Delta_q^2 I_F[\rho]. \end{aligned} \tag{5.11}$$

Cauchy–Schwarz makes the square-bracketed expression nonnegative. It vanishes if and only if the phase chirp is linear, $\partial_q\Phi - \bar{p} = \alpha(q-\bar{q})$. Combining $\Delta_q^2 I_F[\rho] \ge 1$ with $j_0 = 1/(4\pi)$ and $\hbar = 2hj_0 = h/(2\pi)$ from Axiom II gives

$$\begin{aligned} \det\Sigma = \Delta_q^2\Delta_p^2 - \sigma_{qp}^2 \ge (hj_0)^2 = \left(\frac{h}{4\pi}\right)^2 = \frac{\hbar^2}{4} \\ \Longrightarrow \quad \Delta_q\Delta_p \ge \sqrt{\frac{\hbar^2}{4} + \sigma_{qp}^2} \ge \frac{\hbar}{2}. \end{aligned} \tag{5.12}$$

Substituting $\det\Sigma \ge h^2/(16\pi^2)$ into the definition of the dimensionless phase-space state entropy gives

$$\begin{aligned} S_{\text{state}}[\Sigma] &= \frac{1}{2}\ln\left(\frac{(2\pi e)^2\det\Sigma}{h^2}\right) \\ &\ge \frac{1}{2}\ln\left(\frac{4\pi^2e^2}{h^2}\cdot\frac{h^2}{16\pi^2}\right) = \frac{1}{2}\ln\left(\frac{e^2}{4}\right) = 1 - \ln 2 \approx 0.3069 \text{ nat} > 0. \end{aligned} \tag{5.13}$$

For a general d-dimensional multimode system, Williamson's symplectic diagonalization theorem [112] has a physical interpretation: linear coupling and phase-space shear among multiple degrees of freedom can be decoupled into d independent normal phase-space cells by a canonical symplectic transformation that preserves phase-space volume. Each normal mode has a symplectic eigenvalue, its effective single-mode phase-space radius squared, which independently satisfies the minimum information-capacity bound $\nu_a \ge hj_0 = \hbar/2$, $a = 1,\ldots,d$. Similarly, for the d-dimensional

trace variances $\Delta_q^2 = \int_{\mathbb{R}^d} |q-\bar{q}|^2 \rho\, d^d q$ and $\Delta_p^2 = \int_{\mathbb{R}^d} \rho |\nabla\Phi - \bar{p}|^2 d^d q + (\hbar j_0)^2 I_F[\rho]$ defined in Table 3, integrating $\nabla\cdot[(q-\bar{q})\rho] = d\rho + (q-\bar{q})\cdot\nabla\rho$ over $\mathbb{R}^d$ under H1 gives $\int_{\mathbb{R}^d}(q-\bar{q})\cdot\nabla\rho\, d^d q = -d$. Applying the $L^2(\mathbb{R}^d,\mathbb{R}^d)$ vector Cauchy–Schwarz inequality to $\boldsymbol{f}(q) \equiv \sqrt{\rho(q,t)}(q-\bar{q})$ and $\boldsymbol{g}(q) \equiv \sqrt{\rho(q,t)}\nabla\ln\rho(q,t)$, namely $d^2 = \left(\int_{\mathbb{R}^d} \boldsymbol{f}\cdot\boldsymbol{g}\, d^d q\right)^2 \le \left(\int_{\mathbb{R}^d}|\boldsymbol{f}|^2 d^d q\right)\left(\int_{\mathbb{R}^d}|\boldsymbol{g}|^2 d^d q\right) = \Delta_q^2 I_F[\rho]$, yields the d-dimensional Stam–Cramér–Rao bound $\Delta_q^2 I_F[\rho] \ge d^2$ and trace uncertainty relation $\Delta_q\Delta_p \ge d\hbar/2$ (saturated if and only if $\boldsymbol{g} = c\boldsymbol{f}$ and $\nabla\Phi = \bar{p}$, i.e., for an isotropic d-dimensional Gaussian). The multimode determinant bound, trace uncertainty bound, covariance-entropy bound, and Gaussian-state purity formula $\mu_{\mathrm{pur}}^{(\mathrm{Gauss})} = \mathrm{Tr}(\hat{\rho}^2)$ are therefore

$$\det\Sigma = \prod_{a=1}^{d}\nu_a^2 \ge \left(\frac{\hbar}{2}\right)^{2d}, \qquad \Delta_q\Delta_p \ge d\frac{\hbar}{2}, \qquad S_{\mathrm{state}}^{(d)}[\Sigma] \ge d(1-\ln 2), \\ \mu_{\mathrm{pur}}^{(\mathrm{Gauss})} = \frac{(\hbar/2)^d}{\sqrt{\det\Sigma}}. \tag{5.14}$$

This completes the proof of Theorem 3. □

5.2 Theorem 4: postulated channel-resolution time scale for a reference information increment

Theorem 4 (Postulated channel-resolution time scale)**.** Consider an elementary diffusive measurement channel governed by Axiom III with pure entanglement interaction energy spread $\Delta E_{\mathrm{int}}(t)$. To extract a target mutual-information increment $\Delta I_{\mathrm{ex}} \ge \Delta I_{\mathrm{target}} > 0$ over the time window $[t_0, t_0+\Delta t]$, the time-averaged interaction energy spread $\overline{\Delta E}_{\mathrm{int}} \equiv (\Delta t)^{-1}\int_{t_0}^{t_0+\Delta t}\Delta E_{\mathrm{int}}(t)\, dt$ and duration Δt must satisfy $\overline{\Delta E}_{\mathrm{int}}\Delta t \ge h\,\Delta I_{\mathrm{target}}$. In particular, for the reference resolution unit $\Delta I_{\mathrm{target}} = j = 1/(4\pi)$ nat of Axiom II (at constant or upper-bounded ΔE_{int}), the required channel duration obeys

$$\Delta E_{\mathrm{int}}\Delta t \ge h j_0 = \frac{\hbar}{2}, \qquad \Delta t_{\min}(j) = \frac{j}{C_{\max}} = \frac{\hbar}{2\Delta E_{\mathrm{int}}}. \tag{5.15}$$

Remark (Physical interpretation and scope with respect to measurement priors). Rather than representing a kinematic commutator relation for a time operator, Eq. (5.15) is a direct dynamical corollary of combining the reference scale $j = 1/(4\pi)$ nat (Axiom II) with the channel-capacity postulate $C_{\max} = \Delta E_{\mathrm{int}}/h$ (Axiom III). In the baseline Markovian dynamics of Axiom III, information is extracted continuously, so Eq. (5.15) gives the minimum time required to accumulate the specific reference increment $\Delta I_{\mathrm{ex}} = j$ (corresponding to a Gaussian pointer compression ratio $\Delta_M(t)/\Delta_M(0) = e^{-j}$ or expected Bures diffusion $\mathbb{E}[\int ds_{\mathrm{QFI}}^2] = 2j$). For discrete QND branch discrimination with prior weights $\boldsymbol{P}_0$, Theorem 7 [Eq. (5.34)] shows that complete asymptotic discrimination extracts total Shannon mutual information $H(\boldsymbol{P}_0)$; hence $\Delta I_{\mathrm{ex}} \ge j$ holds whenever the extracted Shannon information satisfies $\Delta I_{\mathrm{Sh}} \ge j$ (such as symmetric binary readout where $H_b(1/2) = \ln 2 \approx 8.71\, j > j$, whereas a highly skewed binary prior with $p = 10^{-3}$ has $H_b(p) \approx 0.0079\,\mathrm{nat} < j$ and therefore requires only $\Delta t \ge h H_b(p)/\Delta E_{\mathrm{int}}$ in the baseline Markovian model). Converting $\Delta t_{\min}(j)$ into a prior-independent hard commitment gate requires the additional non-Markovian hypothesis Assumption H3 (Section 8.2).

Proof. Integrating the Nyquist–Planck capacity bound (3.7) of Axiom III over $[t_0, t_0+\Delta t]$ bounds the accumulated mutual information ΔI_{ex}:

$$\Delta I_{\mathrm{ex}} = \int_{t_0}^{t_0+\Delta t}\gamma_{\mathrm{info}}(t)\, dt \le \int_{t_0}^{t_0+\Delta t} C_{\max}(t)\, dt = \frac{\overline{\Delta E}_{\mathrm{int}}\Delta t}{h}\ \mathrm{nat}. \tag{5.16}$$

Setting $\Delta I_{\mathrm{ex}} \ge \Delta I_{\mathrm{target}} = j = j_0$ nat from Axiom II, Eq. (3.5), and multiplying by h gives

$$\overline{\Delta E}_{\mathrm{int}}\Delta t \ge h j_0 = \frac{h}{4\pi} = \frac{\hbar}{2}, \tag{5.17}$$

which yields Eq. (5.15). □

Remark (Role as an axiomatic corollary and comparison with quantum speed limits). Mathematically, Theorem 4 is a direct algebraic corollary of combining the reference resolution step $j = 1/(4\pi)$ nat of Axiom II with the capacity postulate $C_{\max} = \Delta E_{\text{int}}/h$ of Axiom III. Physically, it is instructive to distinguish the operational tasks behind three characteristic time scales at the same energy spread ΔE_{int} [86–90]: (i) $\Delta t_{\min}(j) = \hbar/(2\Delta E_{\text{int}})$ is the minimum channel duration required to extract one elementary resolution unit $j = 1/(4\pi)$ nat ≈ 0.0796 nat; (ii) the Mandelstam–Tamm (MT) orthogonalization time $\tau_{\text{MT}} = \pi\hbar/(2\Delta E_{\text{int}})$ [86] is the time required for closed unitary evolution to rotate a pure state through the geodesic orthogonality angle $\pi/2$; and (iii) $\tau_{\text{Nyq}}(\ln 2) = (h \ln 2)/\Delta E_{\text{int}} = (2\pi \ln 2)\hbar/\Delta E_{\text{int}} \simeq 4.355\hbar/\Delta E_{\text{int}}$ is the channel time required under Axiom III to extract the full Shannon mutual information $\Delta I_{\text{ex}} = \ln 2$ nat ≈ 0.6931 nat needed for complete binary discrimination. At a common energy scale ΔE_{int}, these distinct operational thresholds obey the ordering

$$\Delta t_{\min}(j) = \frac{\hbar}{2\Delta E_{\text{int}}} < \tau_{\text{MT}} = \frac{\pi\hbar}{2\Delta E_{\text{int}}} < \tau_{\text{Nyq}}(\ln 2) = \frac{2\pi \ln 2 \cdot \hbar}{\Delta E_{\text{int}}}. \tag{5.18}$$

5.3 Theorem 5: Schrödinger equivalence in the closed limit and dynamics of the uncertainty function

Theorem 5 (Schrödinger-equivalent mapping in the closed limit and dynamics of the uncertainty function)**.** In the closed limit without coupling to a measurement channel, $\Delta E_{\text{int}} = 0 \Rightarrow \gamma_{\text{info}} = 0$ and $d\Phi_{\text{back}} = 0$. Under technical condition H1, the following statements hold:

1. The evolution equations (3.8a)–(3.8b) of Axiom III reduce to a closed system of partial differential equations. Under the Kähler complex-coordinate map of Theorem 1, $\psi(q,t) \equiv \sqrt{\rho(q,t)}\exp[(i/\hbar)\Phi(q,t)]$, these equations are necessary and sufficient for the standard linear Schrödinger equation, $i\hbar\partial_t\psi = [-(\hbar^2/2m)\nabla^2 + V]\psi$.

2. For a one-dimensional Gaussian wave packet in an external potential $V(q,t)$ that is at most quadratic, or for a general smooth potential after variational projection onto the Gaussian submanifold, the position mean $\bar{q}(t)$ and the objective uncertainty function $\Delta_q(t)$ satisfy the closed second-order ordinary differential equations of Ermakov–Pinney type:

$$m\frac{d^2\bar{q}}{dt^2} = -\left\langle \frac{\partial V}{\partial q} \right\rangle, \qquad m\frac{d^2\Delta_q}{dt^2} = \frac{\hbar^2}{4m\Delta_q^3} - \Delta_q \left\langle \frac{\partial^2 V}{\partial q^2} \right\rangle. \tag{5.19}$$

Remark (Physical intuition and interpretation: the information origin of the Bohm quantum potential and wave-packet elasticity). In de Broglie–Bohm theory, the quantum potential $Q = -(\hbar^2/2m)(\nabla^2\sqrt{\rho}/\sqrt{\rho})$ in the quantum Hamilton–Jacobi equation appears as a nonlocal force inferred by substituting the complex wave function into the Schrödinger equation. The proof of Theorem 5 identifies its physical role as the Euler–Lagrange variational pressure of the Fisher-information kinetic energy $T_{\text{info}} = [(hj_0)^2/(2m)]\int \rho|\nabla \ln \rho|^2\, d^dq$. This pressure arises from variation with respect to the probability density $\rho(q)$ and is required to maintain spatial information localization; see (5.21).

For a Gaussian packet, this information pressure gives the Ermakov–Pinney equation (5.19). The centroid $\bar{q}(t)$ follows Ehrenfest's form of Newton's second law [113], while the objective position uncertainty $\Delta_q(t)$ obeys nonlinear Ermakov–Pinney breathing dynamics [114]. The first term on the right, $+\hbar^2/(4m\Delta_q^3)$, is the outward repulsive information-expansion force generated by Fisher kinetic energy. It prevents collapse into a geometric singularity of infinite density and drives the dispersive broadening of a free packet. The second term, $-\Delta_q\langle\partial_q^2 V\rangle$, is the inward restoring force

supplied by external-potential curvature. An exact balance between these forces produces a stable quantum ground state.

Proof. **Step 1: direct calculation of the functional derivative $\delta H/\delta\rho$.** Write the Fisher-information integral as $F[\rho] \equiv \int_{\mathbb{R}^d} \rho|\nabla \ln \rho|^2 \, d^dq = \int_{\mathbb{R}^d} (|\nabla\rho|^2/\rho) \, d^dq$. Apply an arbitrary smooth compactly supported variation $\delta\rho(q)$. Take the first variation and integrate by parts using the multidimensional divergence theorem. The boundary terms at infinity vanish by H1, giving

$$\begin{aligned} \delta F[\rho] &= \int_{\mathbb{R}^d} \left[\frac{2\nabla\rho \cdot \nabla(\delta\rho)}{\rho} - \frac{|\nabla\rho|^2}{\rho^2}\delta\rho \right] d^dq \\ &= \int_{\mathbb{R}^d} \left[-\frac{2\nabla^2\rho}{\rho} + \frac{|\nabla\rho|^2}{\rho^2} \right] \delta\rho \, d^dq. \end{aligned} \tag{5.20}$$

Direct differentiation also gives $\nabla^2\sqrt{\rho} = \nabla \cdot [\nabla\rho/(2\sqrt{\rho})] = \nabla^2\rho/(2\sqrt{\rho}) - |\nabla\rho|^2/(4\rho^{3/2})$. Dividing this expression by $\sqrt{\rho}$ and multiplying by -4 gives the identity

$$\frac{\delta F[\rho]}{\delta\rho(q,t)} = -\frac{2\nabla^2\rho}{\rho} + \frac{|\nabla\rho|^2}{\rho^2} \equiv -4\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}. \tag{5.21}$$

Substitute (5.21) into the variational derivative of (3.6). Since $2hj_0 = \hbar$ implies $4(hj_0)^2/(2m) = \hbar^2/(2m)$, Eqs. (3.8a)–(3.8b) become, in the closed limit,

$$\frac{\partial\Phi(q,t)}{\partial t} + \frac{|\nabla\Phi(q,t)|^2}{2m} + V(q,t) - \frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho(q,t)}}{\sqrt{\rho(q,t)}} = 0, \tag{5.22a}$$

$$\frac{\partial\rho(q,t)}{\partial t} + \nabla \cdot \left(\rho(q,t)\frac{\nabla\Phi(q,t)}{m} \right) = 0. \tag{5.22b}$$

Step 2: proof of the Schrödinger equation in Kähler complex coordinates. By Theorem 1, $\psi(q,t) = \sqrt{\rho}\, e^{i\Phi/\hbar}$ is single-valued throughout configuration space and twice smoothly differentiable across a codimension-1 nodal hypersurface. Direct evaluation of $i\hbar\partial_t\psi$ and $[-(\hbar^2/2m)\nabla^2 + V]\psi$ gives

$$\begin{aligned} & i\hbar\frac{\partial\psi}{\partial t} - \left(-\frac{\hbar^2}{2m}\nabla^2 + V \right)\psi \\ &= \left\{ \frac{i\hbar}{2\sqrt{\rho}} \left[\frac{\partial\rho}{\partial t} + \nabla \cdot \left(\rho\frac{\nabla\Phi}{m} \right) \right] \right. \\ &\quad \left. - \sqrt{\rho} \left[\frac{\partial\Phi}{\partial t} + \frac{|\nabla\Phi|^2}{2m} + V - \frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}} \right] \right\} e^{i\Phi/\hbar} = 0. \end{aligned} \tag{5.23}$$

The linear independence of the real and imaginary parts makes (5.22a)–(5.22b) necessary and sufficient for the linear Schrödinger equation $i\hbar\partial_t\psi = [-(\hbar^2/2m)\nabla^2 + V]\psi$.

Step 3: derivation of the second-order evolution equation for the one-dimensional Gaussian uncertainty function $\Delta_q(t)$. Insert the Gaussian density (5.10) into the continuity equation (5.22b). Solving for the flow potential gives $\Phi(q,t) = m\dot{\bar{q}}(q-\bar{q}) + [m\dot{\Delta}_q/(2\Delta_q)](q-\bar{q})^2 + \varphi_0(t)$. The linear term $m\dot{\bar{q}}(q-\bar{q})$ describes rigid translation of the packet at its group velocity. The quadratic chirp $[m\dot{\Delta}_q/(2\Delta_q)](q-\bar{q})^2$ describes the radial breathing flow around the

centroid, $v_{\mathrm{breath}} = (\dot{\Delta}_q/\Delta_q)\delta q$, corresponding to expansion or contraction. Setting $\delta q \equiv q - \bar{q}(t)$ and substituting into (5.22a) gives

$$\begin{aligned} \frac{\partial \Phi}{\partial t} + \frac{1}{2m}\left(\frac{\partial \Phi}{\partial q}\right)^2 &= \left(\dot{\varphi}_0 - \frac{1}{2}m\dot{\bar{q}}^2\right) + m\ddot{\bar{q}}\,\delta q + \frac{m\ddot{\Delta}_q}{2\Delta_q}(\delta q)^2, \\ -\frac{\hbar^2}{2m}\frac{\partial_q^2\sqrt{\rho}}{\sqrt{\rho}} &= \frac{\hbar^2}{4m\Delta_q^2} - \frac{\hbar^2}{8m\Delta_q^4}(\delta q)^2. \end{aligned} \tag{5.24}$$

Comparing the coefficients of $(\delta q)^1$ and $(\delta q)^2$ yields (5.19) for any quadratic potential; for a general smooth potential $V(q,t)$, restricting the symplectic action $S[\rho,\Phi] = \int(\int \Phi\partial_t\rho\, dq - H[\rho,\Phi])dt$ to the Gaussian manifold (5.10) and applying Stein's Gaussian integration-by-parts identities $\partial\langle V\rangle/\partial\bar{q} = \langle\partial_q V\rangle$ and $\partial\langle V\rangle/\partial\Delta_q = \Delta_q\langle\partial_q^2 V\rangle$ yields the exact same variational equations (5.19). Finally, in the controlled semiclassical (WKB) regime where the local de Broglie wavelength $\widetilde{\lambda}_{\mathrm{dB}} = \hbar/|\nabla\Phi|$ is much smaller than the characteristic variation scale L_{env} of ρ and V away from nodal surfaces and caustics, the Fisher quantum potential $-(\hbar^2/2m)\nabla^2\sqrt{\rho}/\sqrt{\rho} = O(\hbar^2/mL_{\mathrm{env}}^2)$ is suppressed by $(\widetilde{\lambda}_{\mathrm{dB}}/L_{\mathrm{env}})^2 \ll 1$ relative to $|\nabla\Phi|^2/(2m)$, and Eqs. (5.22a)–(5.22b) reduce directly to the classical Hamilton–Jacobi equation $\partial_t\Phi + |\nabla\Phi|^2/(2m) + V = 0$ coupled to classical Liouville continuity transport $\partial_t\rho + \nabla\cdot(\rho\nabla\Phi/m) = 0$. This completes the proof of Theorem 5. □

5.4 Theorem 6: harmonic-oscillator spectra, orbital angular-momentum quantization, and the three-dimensional hydrogen spectrum

Theorem 6 (Harmonic-oscillator spectrum, orbital angular-momentum quantization, and bound--state hydrogen spectrum)**.** Direct application of the stationary equation (5.22a) of Theorem 5 together with the Sobolev normalizability, loop-holonomy, and nodal-jump conditions of Axiom I and H1 yields:

1. For the one-dimensional harmonic oscillator $V(q) = \frac{1}{2}m\omega_0^2 q^2$, the ground-state uncertainty $\Delta_{q,0} = \sqrt{\hbar/(2m\omega_0)}$ and the discrete spectrum $E_n = (n+\frac{1}{2})\hbar\omega_0$, $n = 0, 1, 2, \ldots$.
2. For a three-dimensional central potential, azimuthal loop holonomy [Eq. (3.1)] gives $L_z = m_l\hbar$, $m_l \in \mathbb{Z}$, while $H^1(S^2)$ polar regularity gives $L^2 = l(l+1)\hbar^2$, where $l = |m_l| + n_\theta$ and $n_\theta \in \mathbb{N}_0$.
3. For the three-dimensional hydrogen Coulomb potential $V(r) = -e^2/(4\pi\epsilon_0 r)$, the Bohr radius $a_0 = 4\pi\epsilon_0\hbar^2/(me^2)$, ground-state radial standard deviation $\Delta_{r,0} \equiv \sqrt{\langle r^2\rangle - \langle r\rangle^2} = (\sqrt{3}/2)a_0$ (with $\langle r\rangle = \frac{3}{2}a_0$ and three-dimensional position uncertainty $\Delta_{\boldsymbol{q},0} \equiv \sqrt{\langle r^2\rangle} = \sqrt{3}\,a_0$), unique global ground-state energy $E_1 = -me^4/[2(4\pi\epsilon_0)^2\hbar^2] \simeq -13.6$ eV, and complete bound-state ($E < 0$) Rydberg spectrum $E_n = E_1/n^2$, with principal quantum number $n = n_r + l + 1$ and orbital degeneracy n^2 (complemented for $E \geq 0$ by the continuous Coulomb scattering spectrum).

Remark (Physical intuition and interpretation: zero-point energy, atomic stability, and the topology of discrete energy levels). In classical mechanics, a harmonic oscillator may rest at the potential minimum with $E = 0$, whereas an electron orbiting a nucleus radiates electromagnetic waves and falls inward, leading to atomic collapse. QICM provides a common information-capacity account of these phenomena. If the electron probability cloud contracts indefinitely toward the nucleus, $\Delta_r \to 0$, the attractive Coulomb energy decreases as $-e^2/(4\pi\epsilon_0\Delta_r)$, but the Fisher-information kinetic energy required to sustain such localization grows faster, as $+\hbar^2/(2m\Delta_r^2)$. The repulsive information pressure balances Coulomb attraction at the Bohr radius a_0, as expressed by the Bogomol'nyi completion of squares in (5.26), thereby giving matter a stable volume and a nonzero ground-state energy.

The discrete excitation quantum numbers also have a direct topological interpretation. The magnetic quantum number $|m_l|$ counts phase-vortex windings about the polar axis; the polar quantum number n_θ counts nodal cones in the latitudinal direction; and the radial quantum number n_r counts concentric spherical nodal shells. Each additional codimension-1 nodal hypersurface, accompanied by the $\pi\hbar$ phase jump in Axiom I, introduces steeper internal gradients into the probability distribution and raises the Fisher-information kinetic energy in discrete increments, as in $I_F[\rho_n] = (2n+1)/\Delta_{q,0}^2$ in (5.25).

Proof. **Step 1: the one-dimensional oscillator ground state and the n-node excited state.** For the ground state, $n = 0$, set $\ddot{\Delta}_q = 0$ and $\langle \partial_q^2 V \rangle = m\omega_0^2$ in (5.19). This gives $\Delta_{q,0} = \sqrt{\hbar/(2m\omega_0)}$ and $E_0 = \frac{1}{2}\hbar\omega_0$. For an excited state with n first-order nodes, Eq. (3.2) of Axiom I prescribes $\Delta\Phi = \pi\hbar$ across each node. The signed real amplitude $u_n(q) = \pm\sqrt{\rho_n(q)}$ satisfies $-(\hbar^2/2m)u_n'' + \frac{1}{2}m\omega_0^2 q^2 u_n = E_n u_n$. Set $\xi = \sqrt{m\omega_0/\hbar}\, q$ and $u_n(\xi) = H(\xi)e^{-\xi^2/2}$. A power-series expansion gives the recurrence $a_{k+2}/a_k = [2k - (2E_n/(\hbar\omega_0) - 1)]/[(k+1)(k+2)]$. The square-integrability and normalization condition imposed by Axiom I forces termination at a nonnegative integer $k = n$, uniquely fixing the eigenenergy and Fisher information:

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_0, \qquad I_F[\rho_n] = \frac{2n+1}{\Delta_{q,0}^2}, \qquad n = 0, 1, 2, \ldots. \tag{5.25}$$

Step 2: three-dimensional orbital angular-momentum quantization. In spherical coordinates (r, θ, φ), apply the circulation condition (3.1), with $\chi = 0$, to a closed loop about the z axis: $\oint (\partial\Phi/\partial\varphi)\, d\varphi = 2\pi L_z = m_l h \Rightarrow L_z = m_l\hbar$, $m_l \in \mathbb{Z}$. Substitution into (5.22a) and separation of the polar variable $\mu = \cos\theta$ yield the associated Legendre series. The $H^1(S^2)$ finite-Fisher-information condition in Axiom I forces the series to terminate at a nonnegative integer $n_\theta \in \mathbb{N}_0$. Defining $l \equiv |m_l| + n_\theta$ then gives $L^2 = l(l+1)\hbar^2$.

Step 3: uniqueness of the global hydrogen ground state and the complete Rydberg spectrum. For any three-dimensional state (ρ, Φ) satisfying H1, use the divergence identity $\nabla \cdot (\hat{e}_r \rho) = \partial_r \rho + (2/r)\rho$. Integration by parts over all space gives $\int_{\mathbb{R}^3} (2\sqrt{\rho}\, \partial_r \sqrt{\rho})\, d^3q = -\int_{\mathbb{R}^3} (2/r)\rho\, d^3q$. Consequently, in the Coulomb potential $V(r) = -e^2/(4\pi\epsilon_0 r)$, the total energy functional (3.6) admits an exact completion into nonnegative squares. With $a_0 \equiv 4\pi\epsilon_0\hbar^2/(me^2)$ and $E_1 \equiv -\hbar^2/(2ma_0^2) = -me^4/[2(4\pi\epsilon_0)^2\hbar^2]$, this completion reads

$$H[\rho, \Phi] = E_1 + \int_{\mathbb{R}^3} \left[\frac{\rho |\nabla\Phi|^2}{2m} + \frac{\hbar^2}{2m} \left| \nabla\sqrt{\rho} + \frac{\hat{e}_r}{a_0}\sqrt{\rho} \right|^2 \right] d^3q \geq E_1. \tag{5.26}$$

Equality holds if and only if $\nabla\Phi = 0$ and $\nabla\sqrt{\rho} = -(\hat{e}_r/a_0)\sqrt{\rho}$. These conditions give the unique density $\rho_{1,0,0}(r) = (\pi a_0^3)^{-1} e^{-2r/a_0}$, with radial moments $\langle r \rangle = \frac{3}{2}a_0$, $\langle r^2 \rangle = 3a_0^2$, radial standard deviation $\Delta_{r,0} = \sqrt{\langle r^2 \rangle - \langle r \rangle^2} = (\sqrt{3}/2)a_0$, and 3D position uncertainty $\Delta_{\boldsymbol{q},0} = \sqrt{\langle r^2 \rangle} = \sqrt{3}\, a_0$ (satisfying $\Delta_{\boldsymbol{q},0}^2 I_F[\rho_{1,0,0}] = 3a_0^2 \cdot (4/a_0^2) = 12 > 3^2 = 9$). For an excited state with a general angular quantum number l and n_r first-order spherical nodal shells, write the reduced radial real amplitude as $u(r) = rR(r) = r^{l+1}e^{-\kappa r}L(r)$, with $\kappa = \sqrt{-2mE}/\hbar$. Normalization requires its generalized Laguerre series to terminate at $k = n_r \in \mathbb{N}_0$, fixing $\kappa_n = 1/(na_0)$ with $n = n_r + l + 1$. Thus

$$E_n = -\frac{\hbar^2\kappa_n^2}{2m} = \frac{E_1}{n^2}, \qquad g_n = \sum_{l=0}^{n-1} (2l+1) = n^2. \tag{5.27}$$

This completes the proof of Theorem 6. □

5.5 Theorem 7: measurement-collapse dynamics, Born-weight consistency, and the nonselective linear Lindblad master equation

Theorem 7 (Measurement collapse, variance decay, Born-weight consistency, and the nonselective Lindblad equation)**.** Suppose the measurement channel is active, with a deterministic, locally integrable, externally specified bare coupling rate $\kappa_{\mathrm{meas}}(t) > 0$ and the derived information-extraction rate $\gamma_{\mathrm{info}}(t) = 2\kappa_{\mathrm{meas}}(t)\Delta_M^2(t) \le C_{\max}$. Suppose also that condition H2 holds. Under the stochastic information-update equation (3.8a) of Axiom III, the following statements hold for finitely many nondegenerate discrete eigenbranches, $N \ge 2$ under H2a, or for a continuous-spectrum observable $M(q)$ with finite fourth moment under H2b. The moment equations are exact when Hamiltonian transport makes no contribution to the first two moments of M; under the strong-pulse condition H2b alone, they describe the leading measurement-dominated limit. The branch probabilities and discrete Shannon entropy in items 3 and 4 refer to the finite-branch case.

1. **Single-trajectory variance evolution and ensemble variance decay.** The sample variance $\Delta_M^2(t)$ of the measured observable satisfies the following exact Itô stochastic differential equation, which involves the third central moment:

$$\begin{aligned} d(\Delta_M^2(t)) &= -2\gamma_{\mathrm{info}}(t)\Delta_M^2(t)\,dt + \sqrt{2\gamma_{\mathrm{info}}(t)}\,\frac{\mu_3(t)}{\Delta_M(t)}\,dW_t \\ &= -4\kappa_{\mathrm{meas}}(t)\big(\Delta_M^2(t)\big)^2\,dt + 2\sqrt{\kappa_{\mathrm{meas}}(t)}\,\mu_3(t)\,dW_t. \end{aligned} \tag{5.28a}$$

   On an invariant family with $\mu_3(t) \equiv 0$ throughout the measurement interval, and with the Hamiltonian moment contributions absent, the diffusion term in (5.28a) vanishes. A Gaussian posterior preserved by an ideal measurement-only position update is one such family. Its variance obeys the integrated rate law below. Whereas feedback stabilization of γ_{info} alone does not eliminate the stochastic third-moment term proportional to $\mu_3\,dW_t$ for asymmetric states, applying Jensen's inequality $\mathbb{E}[(\Delta_M^2)^2] \ge (\mathbb{E}[\Delta_M^2])^2$ at any deterministic bare coupling rate $\kappa_{\mathrm{meas}}(t)$ yields a universal closed differential inequality for the ensemble variance expectation:

$$\begin{aligned} &\Delta_M^2(t)\Big|_{\mu_3\equiv 0} = \Delta_M^2(0)\exp\left[-2\int_0^t \gamma_{\mathrm{info}}(\tau)\,d\tau\right], \\ &\frac{d}{dt}\mathbb{E}[\Delta_M^2(t)] = -2\mathbb{E}\big[\gamma_{\mathrm{info}}(t)\Delta_M^2(t)\big] \le -4\kappa_{\mathrm{meas}}(t)\big(\mathbb{E}[\Delta_M^2(t)]\big)^2. \end{aligned} \tag{5.28b}$$

2. For finitely many discrete QND branches ($N < \infty$ under H2a), as the accumulated measurement strength diverges, $A(t) = \int_0^t \kappa_{\mathrm{meas}}(\tau)\,d\tau \to \infty$, $\mathbb{E}[\Delta_M^2(\infty)] = 0$ and each individual trajectory converges almost surely within $\mathcal{M}_{\mathrm{ontic}}$ to a single eigenbranch support Ω_k ($P_k(\infty) \in \{0,1\}$; at any finite time $A(t) < \infty$, operational branch selection is defined up to a prescribed residual error tolerance $1 - P_k(t) \le \varepsilon_{\mathrm{tol}}$). For a continuous-spectrum observable $M(q) \in C^1(M)$, the trajectory remains in the regular state space $\mathcal{M}_{\mathrm{ontic}}$ ($I_F[\rho] < \infty$) up to any finite instrumental resolution stopping time $\tau_{\Delta_{\mathrm{det}}} = \inf\{t \ge 0 : \Delta_M(t) \le \Delta_{\mathrm{det}}\}$ with $\Delta_{\mathrm{det}} > 0$, whereas the formal point limit $\Delta_{\mathrm{det}} \to 0$ represents an idealized distributional convergence in rigged Hilbert space outside $H^1(M)$.

3. **Born-weight consistency of unbiased martingale updates.** For N discrete QND branches, because Eq. (3.8a) is constructed as a zero-drift unbiased Bayesian innovation update preserving $\mathbb{E}[P_k(t)] = P_k(0)$, the probability that terminal collapse selects the kth eigenregion Ω_k (outcome m_k) equals its postulated initial propensity measure $P_k(0)$ (which after the premeasurement flow of Theorem 2 equals $|\langle k|\psi(0)\rangle|^2$):

$$\mathbb{P}(\text{measurement outcome is } m_k) = P_k(0) = \int_{\Omega_k} \rho(q,0)\,d^dq. \tag{5.29}$$

4. **Nonselective linear Lindblad dynamics and the Shannon-mutual-information martingale identity.** For the externally specified deterministic bare coupling rate $\kappa_{\mathrm{meas}}(t)$, averaging over measurement records gives the nonselective ensemble two-point density kernel $\varrho(q,q';t)=\mathbb{E}[\psi(q,t)\psi^*(q',t)]$, which obeys a state-independent linear Lindblad–GKSL master equation. For any finite number N of QND branches, the accumulated Shannon mutual information between the branch label and the measurement record equals the expectation of the time integral of $\gamma_{\mathrm{info}}(t)$. This identity concerns discrete Shannon entropy and does not assert a corresponding result for a continuous-spectrum differential entropy.

Remark (Physical intuition and interpretation: bounded martingale competition, unique outcomes, and Born weights). The measurement problem arises because deterministic linear Schrödinger evolution produces an entangled superposition of the system and a macroscopic pointer but does not explain why each experiment yields a unique reading whose frequency is exactly the initial squared amplitude $P_k(0)$. Theorem 7 supplies a microscopic dynamical picture. When the measurement channel opens, the eigenbranch weights $P_k(t)\in[0,1]$ undergo a zero-drift bounded martingale competition driven by measurement innovations [18, 19, 65, 79–82], as in (5.31). Each infinitesimal update in Eq. (3.8a) is postulated as an unbiased Bayesian information correction: because its dt drift vanishes, no branch is systematically favored by the innovation noise, conserving the ensemble expectation $\mathbb{E}[P_k(t)]=P_k(0)$ throughout the evolution (as in standard martingale state-reduction models [65, 66]). Doob's martingale convergence theorem then transports this initial prior measure $P_k(0)$—calibrated for equal-norm branches by the envariance symmetry argument adopted in Section 3.5 [105]—to the terminal outcome frequencies at $t\to\infty$.

At the same time, information extraction continually decreases the total variance $\Delta_M^2(t)$ through the nonpositive drift $-2\gamma_{\mathrm{info}}\Delta_M^2\,dt$ in (5.28a). The distribution concentrates on a single branch until fluctuations cease at the absorbing boundaries $P_k(\infty)\in\{0,1\}$. The terminal weight is either one, indicating selection, or zero, indicating elimination. Because its expectation remains $P_k(0)$, the probability that branch k is ultimately selected must equal $P_k(0)$ exactly—directly analogous to a fair random walk on $[0,1]$ (with absorbing endpoints at 0 and 1 and zero drift) in which a walker starting at position $P_k(0)$ hits the upper boundary 1 with probability $P_k(0)$. In the nonselective ensemble, averaging over records produces an additional exact cancellation: the nonlinear terms involving the single-trajectory mean $\bar{M}_t$ cancel the second-order Itô corrections pointwise in (5.33). The ensemble density kernel consequently obeys a fully linear decoherence equation. This explains how genuine nonunitary collapse in individual experiments coexists with the absence of nonlinear superluminal signaling in ensemble statistics when classical measurement records are not exchanged.

Proof. **Preliminary lemma: stopping-time regularization and boundedness of the normalized measurement kernel.** Define the stopping times $\tau_\varepsilon\equiv\inf\{t\ge0:\Delta_M(t)\le\varepsilon\}$, $\varepsilon>0$, which increase monotonically as $\varepsilon\downarrow0$. On the stochastic interval $[0,\tau_\varepsilon)$, the normalized measurement kernel $\widetilde{M}_t(q)\equiv(M(q)-\bar{M}_t)/\Delta_M(t)$ has unit weighted L^2 norm with respect to $\rho(q,t)\,d^dq$: $\|\widetilde{M}_t\sqrt{\rho}\|_{L^2}^2\equiv1$. Take $\tau_0\equiv\lim_{\varepsilon\to0^+}\tau_\varepsilon$ and extend the diffusion coefficient by zero on the absorbing boundary $t\ge\tau_0$. Itô's lemma can then be applied term by term over the entire time domain.

Step 1: proof of the single-trajectory variance equation (5.28a) **and decay law** (5.28b)**.** Consider the sample first moment $\bar{M}_t\equiv\int_{\mathbb{R}^d}M(q)\rho(q,t)\,d^dq$ and the raw second moment $\mu_2(t)\equiv\int_{\mathbb{R}^d}M^2(q)\rho(q,t)\,d^dq$, so that $\Delta_M^2(t)=\mu_2(t)-\bar{M}_t^2$. Under H2, multiply (3.8a) by $M(q)$ and $M^2(q)$, respectively, and integrate over configuration space. Applying $d(\bar{M}_t^2)=2\bar{M}_t\,d\bar{M}_t+(d\bar{M}_t)^2$ yields

$$\begin{aligned} d\bar{M}_t &= \sqrt{2\gamma_{\mathrm{info}}(t)}\,\Delta_M(t)\,dW_t,\\ d(\bar{M}_t^2) &= 2\sqrt{2\gamma_{\mathrm{info}}(t)}\,\bar{M}_t\Delta_M(t)\,dW_t+2\gamma_{\mathrm{info}}(t)\Delta_M^2(t)\,dt. \end{aligned} \tag{5.30}$$

Subtract $d(\bar{M}_t^2)$ from $d\mu_2(t)$ and use the third-central-moment identity $\int (M^2 - 2\bar{M}_t M)(M - \bar{M}_t)\rho\, d^d q = \int (M - \bar{M}_t)^3 \rho\, d^d q \equiv \mu_3(t)$. This gives (5.28a). If $\mu_3(t) \equiv 0$ is preserved throughout the interval, the diffusion coefficient vanishes and the variance satisfies the pathwise ordinary differential equation $d(\Delta_M^2)/dt = -2\gamma_{\text{info}}(t)\Delta_M^2(t) = -4\kappa_{\text{meas}}(t)(\Delta_M^2(t))^2$. Integrating this relation yields the first identity in (5.28b), where the exponent reflects the accumulated (generally state-dependent) information extraction $\int_0^t \gamma_{\text{info}}(\tau)\, d\tau$; for a deterministic bare rate $\kappa_{\text{meas}}(t)$ and initial variance $\Delta_M^2(0)$, integrating $d(\Delta_M^2)/dt = -4\kappa_{\text{meas}}(t)(\Delta_M^2)^2$ directly gives the algebraic decay law

$$\Delta_M^2(t)\big|_{\mu_3 \equiv 0} = \frac{\Delta_M^2(0)}{1 + 4\Delta_M^2(0) \int_0^t \kappa_{\text{meas}}(\tau)\, d\tau},$$

which reduces to $\Delta_M^2(t) = \Delta_M^2(0)/[1 + 4\kappa_{\text{meas}}\Delta_M^2(0)t]$ at constant κ_{meas}. Sustaining a pure exponential decay $\Delta_M^2(t) = \Delta_M^2(0)e^{-2\gamma_* t}$ within the $\mu_3 \equiv 0$ family requires an adaptive bare rate $\kappa_{\text{meas}}(t) = \gamma_*/[2\Delta_M^2(t)]$ that holds $\gamma_{\text{info}}(t) = \gamma_*$ constant without introducing extra variance drift (a feedback-controlled regime distinct from the deterministic-coupling ensemble Lindblad dynamics derived in Step 3). For a general non-Gaussian or discrete state, take expectations in (5.28a) and apply Jensen's inequality to the convex function $x \mapsto x^2$, namely $\mathbb{E}[(\Delta_M^2)^2] \geq (\mathbb{E}[\Delta_M^2])^2$. This yields the second relation in (5.28b). Integrating that inequality gives the bound $\mathbb{E}[\Delta_M^2(t)] \leq \Delta_M^2(0)/[1 + 4\Delta_M^2(0) \int_0^t \kappa_{\text{meas}}(\tau)\, d\tau]$.

Step 2: Born-weight consistency from Doob's supermartingale convergence theorem, bounded martingale conservation, and an explicit Bayesian solution. Equation (5.28a) makes $\Delta_M^2(t) \geq 0$ a nonnegative continuous-time supermartingale. If $\int_0^t \kappa_{\text{meas}}(\tau)\, d\tau \to \infty$, the bound in (5.28b) and Fatou's lemma imply $\Delta_M^2(\infty) = 0$ with probability one. For any eigenvalue submanifold Ω_k, $k \in \{1, \ldots, N\}$, integration of (3.8a) over Ω_k under H2a gives

$$\begin{aligned} dP_k(t) &= \sqrt{2\gamma_{\text{info}}(t)} \left(\frac{m_k - \bar{M}_t}{\Delta_M(t)} \right) P_k(t)\, dW_t \\ &= 2\sqrt{\kappa_{\text{meas}}(t)}(m_k - \bar{M}_t) P_k(t)\, dW_t. \end{aligned} \tag{5.31}$$

Introduce the accumulated measurement strength $A(t) \equiv \int_0^t \kappa_{\text{meas}}(s)\, ds$ and the readout process $dZ_t = 2\kappa_{\text{meas}}(t)\bar{M}_t\, dt + \sqrt{\kappa_{\text{meas}}(t)}\, dW_t$. Itô's lemma shows that (5.31) admits the explicit Bayesian solution

$$P_k(t) = \frac{P_k(0) \exp[2m_k Z_t - 2m_k^2 A(t)]}{\sum_{j=1}^N P_j(0) \exp[2m_j Z_t - 2m_j^2 A(t)]},$$

which remains positive throughout the time domain and has no finite-time explosion. Since (5.31) has no dt drift, each $P_k(t) \in [0, 1]$ is a bounded continuous-time martingale with conserved expectation $\mathbb{E}[P_k(t)] = P_k(0)$. The terminal condition $\Delta_M^2(\infty) = \sum_{k=1}^N (m_k - \bar{M}_\infty)^2 P_k(\infty) = 0$ forces $P_k(\infty) \in \{0, 1\}$. Doob's bounded martingale convergence theorem [115] and dominated convergence then give

$$\begin{aligned} P_k(0) &= \lim_{t \to \infty} \mathbb{E}[P_k(t)] = \mathbb{E}[P_k(\infty)] \\ &= 1 \cdot \mathbb{P}(P_k(\infty) = 1) + 0 \cdot \mathbb{P}(P_k(\infty) = 0) = \mathbb{P}(P_k(\infty) = 1). \end{aligned} \tag{5.32}$$

Step 3: the nonselective linear Lindblad master equation and the finite-branch Shannon-mutual-information martingale identity.

(1) Exact linearization of the nonselective ensemble. Substitute $\gamma_{\mathrm{info}}(t) = 2\kappa_{\mathrm{meas}}(t)\Delta_M^2(t)$ into (3.8a). For the complex encoding section

$$\psi(q,t) = \sqrt{\rho(q,t)}e^{i\Phi(q,t)/\hbar},$$

apply the second-order Itô formula $d\sqrt{\rho} = (2\sqrt{\rho})^{-1}d\rho - (8\rho^{3/2})^{-1}(d\rho)^2$ on the open support $U_t = \{q \in M \mid \rho(q,t) > 0\}$ (since the multiplicative noise $(M(q) - \bar{M}_t)\rho\, dW_t$ vanishes proportionally to $u(q,t) = \pm\sqrt{\rho}$ at nodal surfaces and preserves the $\pi\hbar$ jump condition (3.2), the evolution extends smoothly across first-order nodes in terms of the signed amplitude u), together with the identity (5.23) of Theorem 5. The resulting single-trajectory stochastic Schrödinger equation is

$$\begin{aligned} d\psi(q,t) = &\left[-\frac{i}{\hbar}\hat{H}\psi - \frac{\kappa_{\mathrm{meas}}}{2}(M(q) - \bar{M}_t)^2\psi\right]dt \\ &+ \sqrt{\kappa_{\mathrm{meas}}}(M(q) - \bar{M}_t)\psi\, dW_t. \end{aligned}$$

Apply the Itô product rule $d(\psi\psi^*) = (d\psi)\psi^* + \psi(d\psi^*) + (d\psi)(d\psi^*)$ to the two-point product $\psi(q,t)\psi^*(q',t)$. The coefficient of the measurement contribution to its dt drift is

$$\begin{aligned} &\kappa_{\mathrm{meas}}\Big[-\frac{1}{2}(M(q) - \bar{M}_t)^2 - \frac{1}{2}(M(q') - \bar{M}_t)^2 \\ &\qquad + (M(q) - \bar{M}_t)(M(q') - \bar{M}_t)\Big] \\ &= -\frac{\kappa_{\mathrm{meas}}}{2}(M(q) - M(q'))^2. \end{aligned}$$

Thus all nonlinear terms involving the state mean $\bar{M}_t$ cancel identically on every trajectory. Taking $\varrho(q,q';t) = \mathbb{E}[\psi(q,t)\psi^*(q',t)]$ removes the zero-mean Wiener-noise term and yields the state-independent linear Lindblad–GKSL master equation [98, 99]:

$$\frac{\partial\varrho(q,q';t)}{\partial t} = -\frac{i}{\hbar}[\hat{H},\varrho](q,q';t) - \frac{\kappa_{\mathrm{meas}}(t)}{2}\big(M(q) - M(q')\big)^2\varrho(q,q';t). \tag{5.33}$$

(2) Shannon-mutual-information martingale identity for N discrete branches. Let $K \in \{1,\dots,N\}$ denote the QND branch label, with prior probability $\mathbb{P}(K = k) = P_k(0)$, and let $\mathcal{F}_t$ be the filtration generated by the readout Z_s, $0 \le s \le t$. In the associated observation model,

$$dZ_t = 2\kappa_{\mathrm{meas}}(t)m_K\, dt + \sqrt{\kappa_{\mathrm{meas}}(t)}\, dB_t,$$

where B_t is a Wiener process independent of K. Conditional likelihoods are proportional to $\exp[2m_kZ_t - 2m_k^2A(t)]$. Bayes' rule therefore gives exactly the normalized weights displayed in Step 2: $P_k(t) = \mathbb{P}(K = k \mid \mathcal{F}_t)$. The innovation $dW_t = [dZ_t - 2\kappa_{\mathrm{meas}}(t)\bar{M}_t\, dt]/\sqrt{\kappa_{\mathrm{meas}}(t)}$ is Wiener with respect to $\mathcal{F}_t$, recovering (5.31).

For $\boldsymbol{P} = (P_1,\dots,P_N)$, define the discrete Shannon entropy in nats by $H(\boldsymbol{P}) = -\sum_{k=1}^N P_k\ln P_k$, with $0\ln 0 = 0$. On the interior of the simplex,

$$\frac{\partial H}{\partial P_k} = -(\ln P_k + 1), \qquad \frac{\partial^2 H}{\partial P_j\partial P_k} = -\frac{\delta_{jk}}{P_k}.$$

Substituting (5.31) into the multivariable Itô formula and using $\sum_k dP_k = 0$ gives

$$\begin{aligned} dH(\boldsymbol{P}_t) &= -\sum_{k=1}^{N} \ln P_k \, dP_k - \frac{1}{2}\sum_{k=1}^{N} \frac{(dP_k)^2}{P_k} \\ &= -2\sqrt{\kappa_{\mathrm{meas}}(t)} \sum_{k=1}^{N} P_k(m_k - \bar{M}_t) \ln P_k \, dW_t \\ &\quad - 2\kappa_{\mathrm{meas}}(t) \sum_{k=1}^{N} P_k(m_k - \bar{M}_t)^2 \, dt \\ &= -2\sqrt{\kappa_{\mathrm{meas}}(t)} \sum_{k=1}^{N} P_k(m_k - \bar{M}_t) \ln P_k \, dW_t - \gamma_{\mathrm{info}}(t) \, dt. \end{aligned}$$

For completeness, initially zero-probability branches remain zero and may be omitted from this calculation. For positive initial weights, first stop before any P_k falls below $\varepsilon > 0$ and then let $\varepsilon \downarrow 0$. Since N and the m_k are finite, $P_k \ln P_k$ is bounded and the deterministic coupling is locally integrable; the entropy's stochastic integral is square integrable on every finite time interval. Its expectation vanishes. The bounds $0 \le H \le \ln N$ and $\gamma_{\mathrm{info}} \le C_{\max}$ justify removal of the localization by dominated convergence, including at the absorbing zero-variance boundary.

In the binary case $P_1 = p$ and $P_2 = 1 - p$, choosing the innovation orientation gives $dp_t = \sqrt{2\gamma_{\mathrm{info}}(t) p_t(1-p_t)}\, dW_t$. The entropy reduces to $H_b(p) = -p \ln p - (1-p)\ln(1-p)$, with $H_b''(p) = -1/[p(1-p)]$. Its Itô correction is explicitly

$$\frac{1}{2} H_b''(p_t)(dp_t)^2 = -\frac{2\gamma_{\mathrm{info}}(t) p_t(1-p_t)}{2p_t(1-p_t)}\, dt = -\gamma_{\mathrm{info}}(t)\, dt,$$

in agreement with the general N-branch calculation.

Write $\tau_{\mathrm{abs}} = \inf\{t \ge 0 : \Delta_M^2(t) = 0\}$, allowing $\tau_{\mathrm{abs}} = \infty$, and extend the measurement update by zero after absorption. The conditional-entropy definition of mutual information and the preceding expectation identity give, for every finite t,

$$\begin{aligned} I_{\mathrm{Sh}}(K; \mathcal{F}_{t\wedge\tau_{\mathrm{abs}}}) &= H(\boldsymbol{P}_0) - \mathbb{E}\big[H(\boldsymbol{P}_{t\wedge\tau_{\mathrm{abs}}})\big] \\ &= \mathbb{E}\left[\int_0^{t\wedge\tau_{\mathrm{abs}}} \gamma_{\mathrm{info}}(\tau)\, d\tau\right] \le C_{\max}\mathbb{E}[t \wedge \tau_{\mathrm{abs}}]. \end{aligned} \tag{5.34}$$

The same identity holds at any almost surely finite readout stopping time τ with $\mathbb{E}[\tau] < \infty$. Apply the stopped entropy calculation first at $T \wedge \tau$, then let $T \to \infty$: boundedness of H permits dominated convergence, while nonnegativity of γ_{info} permits monotone convergence. With the same absorption convention,

$$\begin{aligned} I_{\mathrm{Sh}}(K; \mathcal{F}_\tau) &= H(\boldsymbol{P}_0) - \mathbb{E}[H(\boldsymbol{P}_\tau)] \\ &= \mathbb{E}\left[\int_0^{\tau} \gamma_{\mathrm{info}}(s)\, ds\right] \le C_{\max}\mathbb{E}[\tau]. \end{aligned}$$

Here the bound on the rate is pathwise, whereas both the extracted Shannon information and the completion-time bound are ensemble quantities. This completes the proof of Theorem 7. □

Remark (Numerical example of bounded-martingale collapse). To illustrate the interplay between single-trajectory collapse and ensemble Born-weight conservation in Eqs. (5.31)–(5.33), consider a two-outcome QND measurement ($m_1 = +1$, $m_2 = -1$) with initial prior weight $P_1(0) = 0.65$ and integrated coupling strength $A(\tau_{\max}) = \int_0^{\tau_{\max}} \kappa_{\mathrm{meas}}(s)\, ds = 2.5$, where the conditional readout variable satisfies $Z_{\tau_{\max}} \mid (K = k) \sim \mathcal{N}(2m_k A, A)$. Under the exact Bayesian log-likelihood update

$P_1(\tau_{\max}) = [1 + \exp(-\ln(13/7) - 4Z_{\tau_{\max}})]^{-1}$, individual trajectories concentrate near 1 or 0; the population-averaged residual variance drops from $\Delta_M^2(0) = 4P_1(0)[1 - P_1(0)] = 0.910$ to $\mathbb{E}[\Delta_M^2] \approx 0.0124$ at $A = 1.73$ and $\mathbb{E}[\Delta_M^2(\tau_{\max})] \approx 0.00229$ at $A = 2.5$, while the ensemble mean remains conserved at $\mathbb{E}[P_1(t)] = P_1(0) = 0.650$ for all $t \geq 0$ (up to standard binomial sampling error $\sqrt{P_1(0)[1 - P_1(0)]/N_{\mathrm{MC}}}$ in a finite sample of N_{MC} runs).

6 Physical reconstruction of characteristic quantum experiments

The results of Sections 4 and 5 give quantitative physical accounts of double-slit duality, EPR–Bell violations, and the quantum Zeno effect.

6.1 Double-slit interference and the conditional visibility–predictability relation $V^2 + D^2 \leq 1$

Let a particle of mass m and longitudinal momentum p_x pass through slits separated by d, at $y_1 = +d/2$ and $y_2 = -d/2$.

1. **Interference without path detection and osmotic guidance.** For $\gamma_{\mathrm{info}} = 0$, Theorem 5 gives the coherent density at a far-field distance $L \gg d$ from branches (ρ_1, Φ_1) and (ρ_2, Φ_2):

$$\begin{aligned} \rho(y) &= \rho_1(y) + \rho_2(y) + 2\sqrt{\rho_1(y)\rho_2(y)}\cos\left(\frac{\Delta\Phi_{21}(y)}{\hbar}\right), \\ \Delta\Phi_{21}(y) &\equiv \Phi_2(y) - \Phi_1(y) = \frac{p_x d}{L} y. \end{aligned} \tag{6.1}$$

   At dark fringes, $\Delta\Phi_{21} = (2n+1)\pi\hbar$ and $\rho(y) \to 0$: the local osmotic momentum $p_{\mathrm{osm}} = -(\hbar/2)\partial_y \ln\rho \propto (y - y_n)^{-1}$ diverges across each nodal line, while for finite-width slit packets the Fisher quantum potential $Q = -(\hbar^2/2m)(\partial_y^2\sqrt{\rho}/\sqrt{\rho})$ in Eq. (5.22a) develops repulsive barriers at the dark-fringe valleys (whereas for an idealized plane standing wave $\sqrt{\rho} \propto |\cos(ky)|$, $Q = \hbar^2 k^2/(2m)$ is spatially constant on each open nodal interval), guiding probability flux toward the bright fringes separated by $\Delta y = hL/(p_x d)$.

2. **Path information and fringe erasure.** Measure $M \in \{+1, -1\}$ with symmetric initial weights $P_1(0) = P_2(0) = 1/2$ and $\Delta_M(0) = 1$. After an interaction time τ, Theorem 7 gives conditional weights $P_1(\tau), P_2(\tau)$. Define conditional branch predictability $D(\tau) \equiv |P_1(\tau) - P_2(\tau)|$ and conditional visibility $V(\tau) \equiv 2\sqrt{P_1(\tau)P_2(\tau)} = \Delta_M(\tau)$. Normalization and Cauchy–Schwarz give

$$(P_1 + P_2)^2 = 1, \qquad \mathbb{E}[V]^2 \leq \mathbb{E}[V^2] = 1 - \mathbb{E}[D^2] \leq 1 - \mathbb{E}[D]^2.$$

   Hence, along each conditional pure-state trajectory, the conditional visibility and branch predictability satisfy the exact identity $V^2(\tau) + D^2(\tau) = 4P_1(\tau)P_2(\tau) + [P_1(\tau) - P_2(\tau)]^2 = [P_1(\tau) + P_2(\tau)]^2 = 1$, and Cauchy–Schwarz gives the following inequality for averages over conditional trajectories. This is a conditional analogue of a duality bound: the average conditional visibility and branch predictability used here need not equal the unread fringe visibility and optimized detector-based distinguishability in Englert's operational inequality [116]. Explicitly,

$$V^2(\tau) + D^2(\tau) = 1 \quad \text{(conditional trajectory)}, \qquad \mathbb{E}[V]^2 + \mathbb{E}[D]^2 \leq 1 \quad \text{(ensemble)}. \tag{6.2}$$

   Within this conditional pure-state description, Eq. (6.2) admits an information-geometric interpretation as a Pythagorean allocation between the local branch-weight difference D and residual interbranch uncertainty $\Delta_M = V$.

Remark (Physical intuition and interpretation: how path detection erases interference). Rather than attributing fringe loss solely to classical momentum kicks, Eqs. (6.1)–(6.2) provide a continuous

hydro-informational mechanism: when the slit weights are balanced ($P_1 = P_2 = 1/2$), destructive interference drives $\rho \to 0$ at the dark fringes, creating sharp Fisher osmotic barriers that channel probability flux into the bright fringes. Opening a which-path channel drives the conditional branch weights toward an asymmetric limit (such as $P_1(\tau) \to 1$, $P_2(\tau) \to 0$), preventing complete destructive cancellation in the dark fringes, smoothing out the osmotic waveguide barriers, and relaxing the far-field pattern into a single-slit diffraction envelope.

6.2 Continuous-variable EPR states and CHSH violations

Consider spatially separated equal-frequency oscillators A, B in a two-mode squeezed EPR state with $\Phi_{AB} = 0$. Its normalized objective joint density is

$$\rho_{AB}(q_A, q_B) = \frac{1}{2\pi\Delta_0^2} \exp\left[-\frac{e^{+2r}(q_A - q_B)^2}{4\Delta_0^2} - \frac{e^{-2r}(q_A + q_B)^2}{4\Delta_0^2}\right], \tag{6.3}$$

where $r > 0$ and $\Delta_0 = \sqrt{\hbar/(2m\omega_0)}$.

1. **Nonseparable Fisher information and negative conditional covariance entropy.** The logarithmic gradient is $\partial_{q_A} \ln \rho_{AB} = -(q_A \cosh 2r - q_B \sinh 2r)/\Delta_0^2$. The local Fisher-information force at A depends on q_B, with nonzero coefficient $\sinh 2r$. The covariance entropies obey

$$\begin{aligned} S^{(2)}_{\text{state}}(A, B) &= 2(1 - \ln 2) \\ &< S_{\text{state}}(A) + S_{\text{state}}(B) = 2(1 - \ln 2 + \ln\cosh 2r), \\ S_{\text{state}}(A|B) &= (1 - \ln 2) - \ln\cosh(2r). \end{aligned}$$

   The conditional Gaussian-reference differential entropy $S_{\text{state}}(A|B)$ becomes negative for $r > \frac{1}{2}\operatorname{arcosh}(e/2) \approx 0.411$, reflecting continuous phase-space squeezing of the relative coordinate $q_A - q_B$ below the single-mode Planck cell $\hbar/2$ (distinct from discrete Shannon conditional entropy, which is always nonnegative).

2. **Displaced parity and the spin-singlet CHSH correlation.** Using the premeasurement isometry of Theorem 2, part 3, implement the Banaszek–Wódkiewicz displaced-parity scheme [117]: at dimensionless phase-space displacements α, β, the two-mode parity correlation in the high-squeezing limit $r \gg 1$ depends on the rescaled displacement separation $J \equiv 2e^{2r}|\alpha - \beta|^2 \geq 0$ as $E(\alpha, \beta) \simeq \exp(-2e^{2r}|\alpha - \beta|^2)$. Choosing coplanar displacements of step $\sqrt{J}$ yields the CHSH combination $B_{\text{CHSH}}(J) = 1 + 2e^{-J} - e^{-4J}$ [117], whose derivative $dB_{\text{CHSH}}/dJ = -2e^{-J} + 4e^{-4J}$ vanishes at $e^{3J^*} = 2$, i.e., $J^* = \frac{1}{3}\ln 2$, yielding $B_{\text{CHSH}}(J^*) = 1 + 2 \cdot 2^{-1/3} - 2^{-4/3} = 1 + 3/2^{4/3} \approx 2.19055 > 2$. Likewise, for the spin-1/2 singlet state of Theorem 9, standard Born-rule spin projections at relative angle $\theta_{ab} = \pi/4$ reproduce Tsirelson's bound [34–36, 40–44, 108, 118], which QICM interprets in configuration space through conditional updating of the nonfactorizable joint density:

$$\begin{aligned} B^{\max}_{\text{CHSH}} &= 1 + \frac{3}{2^{4/3}} \approx 2.1905 > 2, \\ B^{\text{spin}}_{\text{CHSH}}\big|_{\theta_{ab}=\pi/4} &= 4\cos\left(\frac{\pi}{4}\right) = 2\sqrt{2} \approx 2.8284 > 2. \end{aligned} \tag{6.4}$$

Remark (Physical intuition and interpretation: the Heptalemma and Jarrett decomposition). The CHSH bound $|S| \leq 2$ assumes Bell factorization, $P(A, B|a, b, \lambda) = P(A|a, \lambda)P(B|b, \lambda)$. In the Heptalemma [33], Jarrett's decomposition [51] shows that this is equivalent to parameter independence, $P(B|a, b, \lambda) = P(B|b, \lambda)$, and outcome independence, $P(B|a, b, A, \lambda) = P(B|a, b, \lambda)$. Here $\lambda = (\rho_{AB}, [\Phi_{AB}])$. Although each marginal is broad, with $\Delta^2_{q_A} = \Delta_0^2 \cosh 2r$, the joint density

is compressed into a ridge of width $\Delta_0 e^{-r}$ along $q_A - q_B$ and is nonfactorizable. A recorded result A, encoded by $dW_t^{(A)}$, conditionally slices the shared ridge, violating outcome independence: $P(B|a,b,A,\lambda) \neq P(B|a,b,\lambda)$. Subsequent comparison of the records reveals correlations beyond 2, up to $2\sqrt{2}$.

Without communicating $dW_t^{(A)}$ through a subluminal classical channel, the observer at B averages over A. Positive and negative conditional shifts cancel by bounded-martingale expectation conservation, Eq. (7.11). Parameter independence and measurement independence, $P(\lambda|a,b) = P(\lambda)$, are maintained; the marginal density and Fisher information at B are unaffected. Nonlocal conditional correlations and relativistic causal no-signaling are thus reconciled within a single objective world.

6.3 The Misra–Sudarshan Zeno expansion and finite-bandwidth limits

Consider a double-well tunnelling or coupled two-level system with supports Ω_1, Ω_2 and coupling-energy uncertainty ΔE. Initially $P_1(0) = 1$, $P_2(0) = 0$. During $\delta t = T/N$, the second-order expansion of Theorem 5 gives the Misra–Sudarshan escape probability [119], $P_2(\delta t) = (\Delta E \delta t/\hbar)^2 + O(\delta t^3)$, which grows quadratically, whereas a classical Markov jump has linear growth. After N strong measurements reset the small escaped tail to Ω_1,

$$P_{\text{trans}}(T) = 1 - \left[1 - \left(\frac{\Delta E T}{\hbar N}\right)^2\right]^N = \frac{(\Delta E T)^2}{\hbar^2 N} + O(N^{-2}). \tag{6.5}$$

Remark (Physical intuition and interpretation: finite-bandwidth limits on Zeno freezing). Instantaneous projection assumes $\tau_{\text{pulse}} = 0$ and formally permits $N \to \infty$, $P_{\text{trans}} \to 0$, without an unbounded probe-energy cost. Under the finite-bandwidth constraint of Axiom III, if each stroboscopic reset pulse is required to extract at least one elementary resolution unit $\Delta I_{\text{ex}} \geq j = 1/(4\pi)$ nat (or under the non-Markovian commitment threshold of Assumption H3), Theorem 4 requires a minimum pulse duration $\tau_{\text{pulse}} \geq \Delta t_{\min}(j) = \hbar/(2\Delta E_{\text{int}})$. Within a total interval T, the number of such discrete reset cycles and the residual transition probability obey

$$N_{\max} \leq \frac{T}{\tau_{\text{pulse}}} \leq \frac{2\Delta E_{\text{int}} T}{\hbar}, \qquad P_{\text{trans}}(T) \geq \frac{(\Delta E)^2 T}{2\hbar \Delta E_{\text{int}}} > 0.$$

Similarly, in the continuous nonselective Markovian monitoring regime of Theorem 7, Eq. (5.33), for a two-state observable $M \in \{+1, -1\}$ ($|m_1 - m_2| = 2$), off-diagonal interbranch coherence is damped at the Lindblad dephasing rate $\Gamma_{\text{deph}} = \frac{1}{2}\kappa_{\text{meas}}(m_1 - m_2)^2 = 2\kappa_{\text{meas}}$. Because the apparatus coupling rate κ_{meas} is bounded on the equal-superposition reference state ($\Delta^2_{M,\text{sup}} = 1$) by $2\kappa_{\text{meas}} \leq C_{\max} = \Delta E_{\text{int}}/h$, the continuous-monitoring Zeno escape probability $P_{\text{trans}}(T) \simeq 2(\Delta E/\hbar)^2 T/\Gamma_{\text{deph}} \geq 2(\Delta E/\hbar)^2 hT/\Delta E_{\text{int}}$ remains strictly positive whenever $\Delta E_{\text{int}} < \infty$. Complete Zeno freezing in either the stroboscopic-reset or continuous-monitoring regime therefore requires $\Delta E_{\text{int}} \to \infty$.

7 Extension theorems for relativistic field theory, spin and statistics, the Compton cutoff, and many-body no-signaling

We extend the nonrelativistic single-particle configuration space to four-dimensional Minkowski spacetime, infinite-dimensional classical field configuration space, the intrinsic orientation-frame bundle $\mathbb{R}^3 \times SO(3)$, and the permutation quotient manifold M_N for identical particles. In these settings, we establish and prove Theorems 8–11.

7.1 Theorem 8: Lorentz-covariant reconstruction of free scalar and electromagnetic fields and the number-state–nodal-topology correspondence

Theorem 8 (Lorentz-covariant Madelung–Klein–Gordon correspondence and functional field nodal structure)**.**

1. **Lorentz-covariant Madelung–Klein–Gordon equations and conserved four-current.** In four-dimensional Minkowski spacetime $\mathbb{R}^{1,3}$, with metric $\eta_{\mu\nu} = \mathrm{diag}(+1,-1,-1,-1)$ and d'Alembertian $\Box \equiv \eta^{\mu\nu}\partial_\mu\partial_\nu$, represent a complex scalar field $\varphi(x) \equiv \sqrt{\varrho(x)}\exp(i\Phi(x)/\hbar)$ by a Lorentz-scalar amplitude modulus $\varrho(x) \geq 0$ and a compact action-phase field $[\Phi(x)] \in \mathbb{R}/h\mathbb{Z}$. Define the conserved Klein–Gordon four-current by $J^\mu(x) \equiv -(\varrho(x)/m)\eta^{\mu\nu}\partial_\nu\Phi(x)$. The Lorentz-invariant spacetime action–Fisher functional is

$$A_{\mathrm{rel}}[\varrho,\Phi] = \frac{1}{c}\int_{\mathbb{R}^{1,3}} \varrho(x)\left[\frac{\eta^{\mu\nu}\partial_\mu\Phi\,\partial_\nu\Phi - m^2c^2}{2m} + \frac{(hj_0)^2}{2m}\eta^{\mu\nu}\partial_\mu \ln\varrho\,\partial_\nu\ln\varrho\right] d^4x. \tag{7.1}$$

Stationary variation of Eq. (7.1) yields the continuity equation $\partial_\mu J^\mu = 0$ and the relativistic Madelung–Klein–Gordon equation $\eta^{\mu\nu}\partial_\mu\Phi\,\partial_\nu\Phi - m^2c^2 - \hbar^2(\Box\sqrt{\varrho}/\sqrt{\varrho}) = 0$, which are formally equivalent to the linear Klein–Gordon equation $(\Box + m^2c^2/\hbar^2)\varphi(x) = 0$ [120]. Because the conserved spatial integral is the charge $\int_{\mathbb{R}^3} J^0(x)\,d^3x = -\int_{\mathbb{R}^3}[\varrho(x)/(mc)]\partial_t\Phi(x)\,d^3x$ rather than $\int_{\mathbb{R}^3}\varrho(x)\,d^3x$ (and $J^0(x)$ is guaranteed to be positive only for monochromatic or slowly modulated positive-frequency wavepackets with $-\partial_t\Phi > 0$), a strictly nonnegative ontological probability distribution at arbitrary relativistic energies requires the field-configuration functional representation of Part 2.

2. **Functional dynamics on field configuration space and nodal structure of number-state excitations.** In a fixed Lorentz frame, take the classical field configuration space $C = \{\varphi(\boldsymbol{x}) \in S'(\mathbb{R}^3)\}$ (regulated on a finite volume or ultraviolet lattice with formal flat cylinder measure $D\varphi = \prod_{\boldsymbol{k}} dq_{\boldsymbol{k}}$) as the base manifold. The quantum-field state is specified by a nonnegative objective probability functional $P[\varphi,t] = |\Psi[\varphi,t]|^2 \geq 0$ (normalized by $\int_C D\varphi\,P[\varphi,t] = 1$, with the rigorous infinite-dimensional limit defined via its Radon–Nikodym derivative with respect to the vacuum Gaussian cylinder measure $d\mu_0(\varphi)$ [121]) and a conjugate phase functional $[\Phi[\varphi,t]] \in \mathbb{R}/h\mathbb{Z}$. Its positive-definite Hamiltonian functional is [122]:

$$H_{\mathrm{field}}[P,\Phi] = \int_C D\varphi\,P[\varphi,t]\int_{\mathbb{R}^3} d^3x\left\{\frac{c^2}{2}\left(\frac{\delta\Phi}{\delta\varphi(\boldsymbol{x})}\right)^2 + \frac{(hj_0c)^2}{2}\left(\frac{\delta\ln P}{\delta\varphi(\boldsymbol{x})}\right)^2 + \frac{1}{2}|\nabla\varphi|^2 + \frac{m^2c^2}{2\hbar^2}\varphi^2\right\}, \tag{7.2}$$

where $(hj_0c)^2/2 = \hbar^2c^2/8 \equiv \hbar^2/(8M_{\mathrm{mode}})$ with $hj_0 = \hbar/2$ and mode inertial parameter $M_{\mathrm{mode}} = c^{-2}$. Similarly, on the Coulomb-gauge electromagnetic configuration space $C_{\mathrm{EM}} = \{\boldsymbol{A}_\perp(\boldsymbol{x}) : \nabla\cdot\boldsymbol{A}_\perp = 0\}$, replacing $\delta/\delta\varphi$ by $\epsilon_0^{-1/2}\delta/\delta\boldsymbol{A}_\perp$ (representing the conjugate transverse electric field operator $-\epsilon_0^{-1}\hat{\boldsymbol{\Pi}}_\perp = i\hbar\epsilon_0^{-1}\delta/\delta\boldsymbol{A}_\perp$) and the potential density by the magnetic energy density $\frac{1}{2}\mu_0^{-1}|\nabla\times\boldsymbol{A}_\perp|^2$ yields the positive-definite transverse electromagnetic functional $H_{\mathrm{EM}}[P,\Phi] = \int_{C_{\mathrm{EM}}} D\boldsymbol{A}_\perp\,P[\boldsymbol{A}_\perp,t]\int_{\mathbb{R}^3} d^3x\,\{\frac{1}{2\epsilon_0}(\delta\Phi/\delta\boldsymbol{A}_\perp)^2 + \frac{(hj_0)^2}{2\epsilon_0}(\delta\ln P/\delta\boldsymbol{A}_\perp)^2 + \frac{1}{2\mu_0}|\nabla\times\boldsymbol{A}_\perp|^2\}$. Under the orthogonal normal-mode expansion $\varphi(\boldsymbol{x}) = \sum_{\boldsymbol{k}} q_{\boldsymbol{k}}u_{\boldsymbol{k}}(\boldsymbol{x})$, Theorem 6 implies that the $n_{\boldsymbol{k}}$-particle Fock eigenstate of mode $\boldsymbol{k}$ corresponds to a stationary Hermite–Gaussian mode distribution $P_n(q_{\boldsymbol{k}})$ possessing $n_{\boldsymbol{k}}$ codimension-1 nodal hypersurfaces in amplitude space, with

eigenenergy $E_{\boldsymbol{k}} = (n_{\boldsymbol{k}} + 1/2)\hbar\omega_{\boldsymbol{k}}$. Regularization by a Gaussian ultraviolet kernel $K_\Lambda(\boldsymbol{x} - \boldsymbol{y})$ renders the functional Fisher information finite.

Remark (Physical intuition and interpretation: what is a "particle" in quantum field theory?). Reconciling a continuously distributed field with discrete particle counts in a detector is a central ontological problem in quantum field theory (QFT). The functional Schrödinger–information representation in Theorem 8 gives a direct interpretation: the fundamental physical reality in QFT is the *objective field-configuration probability functional* $P[\varphi, t]$ on the classical field configuration space $C = \{\varphi(\boldsymbol{x})\}$. The vacuum $|0\rangle$ is the nodeless Gaussian ground state that minimizes the total field energy functional (7.2) (balancing functional Fisher localization energy against the quadratic field potential energy). In the real standing-wave normal-mode basis $\{q_{\boldsymbol{k}}\}$, the presence of $n_{\boldsymbol{k}}$ excitation quanta in mode $\boldsymbol{k}$ means that the marginal amplitude distribution $P_n(q_{\boldsymbol{k}})$ has acquired $n_{\boldsymbol{k}}$ *codimension-1 Hermite–Gaussian nodal hypersurfaces* (whereas complex superpositions across degenerate modes, such as $(|1,0\rangle + i|0,1\rangle)/\sqrt{2}$, carry codimension-2 phase vortices in the two-mode plane). In particular, for the transverse electromagnetic field, an $n_{\boldsymbol{k},\lambda}$-photon Fock state in wavevector mode $\boldsymbol{k}$ and polarization λ is not a collection of classical point particles moving through space, but a stationary mode distribution $P_{n_{\boldsymbol{k},\lambda}}(A_{\boldsymbol{k},\lambda})$ with $n_{\boldsymbol{k},\lambda}$ nodes along the transverse vector-potential coordinate $A_{\boldsymbol{k},\lambda}$; a localized single-photon click occurs when coupling to a detector atom or superconducting resonator opens a measurement channel (Axiom III) that transfers this discrete mode excitation across the local coupling bond (Theorem 7). Within the standing-wave mode basis, integer excitation counting thus corresponds directly to the nodal count of the field-configuration functional.

Proof. **Step 1: Covariant single-particle variation.** Apply the four-dimensional divergence theorem to the variation of Eq. (7.1) with respect to $\Phi(x)$. For the variation with respect to $\varrho(x)$, use the covariant form in the Minkowski metric $\eta^{\mu\nu}$ of the variational identity in Theorem 5, Eq. (5.21):

$$\frac{\delta}{\delta\varrho}\int \varrho\eta^{\mu\nu}\partial_\mu \ln\varrho\,\partial_\nu\ln\varrho\, d^4x = -4\frac{\Box\sqrt{\varrho}}{\sqrt{\varrho}}.$$

Substitution of $4(hj_0)^2 = \hbar^2$ gives the single-particle covariant equations.

Step 2: Normal-mode decomposition, dimensional scaling, and nodal counting. In Eq. (7.2), the canonical scalar field $\varphi(\boldsymbol{x})$ has dimension $M^{1/2}L^{1/2}T^{-1}$ so that $\frac{1}{2}|\nabla\varphi|^2$ and $\frac{1}{2}(m^2c^2/\hbar^2)\varphi^2$ have energy density dimension $ML^{-1}T^{-2}$, while the functional derivative $\delta\Phi/\delta\varphi(\boldsymbol{x})$ has dimension $M^{1/2}L^{-3/2}$ and $\frac{1}{2}c^2(\delta\Phi/\delta\varphi)^2$ likewise has dimension $ML^{-1}T^{-2}$. Expanding $\varphi(\boldsymbol{x}) = \sum_{\boldsymbol{k}} q_{\boldsymbol{k}}u_{\boldsymbol{k}}(\boldsymbol{x})$ in a real orthonormal spatial basis $\{u_{\boldsymbol{k}}(\boldsymbol{x})\}$ ($\int_{\mathbb{R}^3} u_{\boldsymbol{k}}u_{\boldsymbol{k}'}\,d^3x = \delta_{\boldsymbol{k}\boldsymbol{k}'}$, dimension $L^{-3/2}$) gives mode coordinates $q_{\boldsymbol{k}}$ of dimension $M^{1/2}L^2T^{-1}$. With respect to the flat mode measure $\prod_{\boldsymbol{k}} dq_{\boldsymbol{k}}$, the Hamiltonian functional (7.2) decomposes into a sum of independent normal-mode harmonic oscillators with conjugate momentum operator $-i\hbar\partial/\partial q_{\boldsymbol{k}}$, inertial parameter $M_{\text{mode}} = c^{-2}$ (dimension $L^{-2}T^2$, so $\frac{1}{2}M_{\text{mode}}\omega_{\boldsymbol{k}}^2q_{\boldsymbol{k}}^2$ has dimension of energy ML^2T^{-2}), and dispersion frequency $\omega_{\boldsymbol{k}} = \sqrt{c^2|\boldsymbol{k}|^2 + m^2c^4/\hbar^2}$. Its nodeless ground state is the Gaussian vacuum distribution $P_0[\varphi] \propto \exp(-\hbar^{-1}\sum_{\boldsymbol{k}}(\omega_{\boldsymbol{k}}/c^2)q_{\boldsymbol{k}}^2)$, which defines the reference Gaussian cylinder measure $d\mu_0(\varphi) = P_0[\varphi]D\varphi$ in the infinite-mode limit [121, 122]. Exciting $n_{\boldsymbol{k}}$ quanta in mode $\boldsymbol{k}$ therefore creates $n_{\boldsymbol{k}}$ codimension-1 nodal hypersurfaces in the marginal distribution $P_n(q_{\boldsymbol{k}})$ by Theorem 6, Eq. (5.25). □

7.2 Theorem 9: Half-integer spin, spherical Kähler–Fisher curvature, Pauli information pressure, and a corollary for hydrogen fine structure

Theorem 9 (Half-integer spin coherent-section geometry, Pauli information pressure, and topological compatibility of statistical sectors)**.**

1. **Half-integer holonomy, the coherent-section Kähler–Fisher identity $\langle \boldsymbol{S}^2\rangle = s(s+1)\hbar^2$, and sign reversal under a 2π rotation.** Consider the single-particle configuration principal-bundle manifold $M_{\rm spin} = \mathbb{R}^3 \times SO(3)$, which includes the intrinsic orientation frame of the local probability flow, with $\pi_1(SO(3)) = \mathbb{Z}_2$. Given the elementary angular-momentum scale $s_{\min} = hj_0 = \hbar/2$ of Axiom II, let $s = n_s/2$ denote the dimensionless spin quantum number for an equivariant highest-weight Hopf coherent section of integer fiber weight $n_s = 2s \in \mathbb{N}^+$ over the spin-polarization base manifold $S^2 \cong SU(2)/U(1)$, with maximal longitudinal projection $S_\parallel = n_s s_{\min} = s\hbar$. Pulling back the dimensionless quantum-Fisher metric $ds^2_{\rm QFI} = (2/\hbar)G_{\rm FR}$ of Theorem 1 [Eq. (4.3)] to S^2 along this coherent section yields the spherical metric $F_{ab} = 2s\, g^{(S^2)}_{ab}$, whose trace against the inverse unit-sphere metric is $I^{(s)}_{{\rm QF},S^2} = 4s$, equal to the dimensionless transverse spin variance $4(\Delta S_x^2 + \Delta S_y^2)/\hbar^2$. The nontrivial $\mathbb{Z}_2$ holonomy sector ($\chi = 1/2$) selects odd n_s and half-integer s, whereas even n_s belongs to the trivial sector ($\chi = 0$). On this equivariant coherent section (and by $SU(2)$ Casimir invariance across the $(2s+1)$-dimensional irreducible representation), the expectation of the total spin squared satisfies

$$\begin{aligned} s_{\min} &= hj_0 = \frac{\hbar}{2}, \\ s &= \frac{n_s}{2}, \qquad S_\parallel = n_s s_{\min} = s\hbar, \\ \langle \boldsymbol{S}^2\rangle &= S_\parallel^2 + (hj_0)^2 I^{(s)}_{{\rm QF},S^2} = s^2\hbar^2 + \frac{\hbar^2}{4}(4s) = s(s+1)\hbar^2. \end{aligned} \tag{7.3}$$

   Around a closed 2π rotation loop $C_{2\pi}$ in $SO(3)$ ($2[C_{2\pi}] = 0$), Eq. (3.1) in the sector $\chi([C_{2\pi}]) = 1/2$ yields the half-integer circulation $(k+1/2)h = (2k+1)\pi\hbar$, reproducing the spinor sign reversal $\psi(\boldsymbol{q}, \boldsymbol{\Omega} + 2\pi) = -\psi(\boldsymbol{q}, \boldsymbol{\Omega})$.

2. **The Pauli exclusion principle and repulsive Fisher information pressure for equal spins.** For N identical particles in three spatial dimensions, the orbital permutation quotient manifold is $M_N = (\mathbb{R}^{3N} \setminus \Delta_{\rm diag})/S_N$, where the pairwise coincidence locus $\Delta_{\rm diag} = \{\boldsymbol{q}_a = \boldsymbol{q}_b\}$ has codimension 3 and $\pi_1(M_N) = S_N$. The one-dimensional unitary characters $\mathrm{Hom}(S_N, U(1)) \cong \mathbb{Z}_2$ in Axiom I, Eq. (3.1), define two topological sectors: bosons, with two-particle exchange holonomy $\Delta\Phi_{ab} = 0 \pmod h$, and fermions, with exchange holonomy $\Delta\Phi_{ab} = h/2 = \pi\hbar \pmod h$. To connect these exchange sectors to intrinsic spin, we impose the additional compatibility assumption that exchange and 2π-rotation holonomies satisfy $\Delta\Phi_{ab} \equiv \Delta\Phi_{2\pi} \pmod h$. The Finkelstein–Rubinstein construction for suitable nonlinear field configurations [123] motivates this assumption, but does not establish it for the framed point-particle quotient specified here. In the equal-spin fermionic sector, exchange antisymmetry $\psi(-\boldsymbol{r}_{ab}) = -\psi(\boldsymbol{r}_{ab})$ forces the codimension-3 coincidence locus $\boldsymbol{r}_{ab} = \boldsymbol{q}_a - \boldsymbol{q}_b = 0$ to lie within the zero set of the relative wave function (forming a codimension-1 nodal plane $z_{ab} = 0$ for the real p_z state $m_l = 0$, or a codimension-2 vortex line $x_{ab} = y_{ab} = 0$ for the complex $p_\pm$ states $m_l = \pm 1$). In every p-wave ($l = 1$) channel, the radial amplitude vanishes linearly as $\sqrt{\rho_{\uparrow\uparrow}} \propto r_{ab}$ as $r_{ab} = |\boldsymbol{q}_a - \boldsymbol{q}_b| \to 0$, so the radial component of the relative-motion Fisher kinetic integrand per unit probability (with reduced mass $\mu = m/2$ and relative Fisher coefficient $(hj_0)^2/m = \hbar^2/(8\mu)$) diverges as

$$\begin{aligned} \rho_{\uparrow\uparrow}(\boldsymbol{q}_1, \ldots, \boldsymbol{q}_N)\big|_{\boldsymbol{q}_a = \boldsymbol{q}_b} &\equiv 0, \\ V^{\rm info}_{\rm Pauli,rad}(r_{ab}) &= \frac{(hj_0)^2}{m}\left(\frac{\partial \ln \rho_{\uparrow\uparrow}}{\partial r_{ab}}\right)^2 \\ &\sim \frac{\hbar^2}{m r_{ab}^2} = \frac{\hbar^2}{2\mu r_{ab}^2} \longrightarrow +\infty \end{aligned} \tag{7.4}$$

(acting alongside the $l = 1$ angular kinetic barrier $l(l+1)\hbar^2/(2\mu r_{ab}^2) = \hbar^2/(\mu r_{ab}^2)$ on S^2, which arises from the polar Fisher gradient in the $m_l = 0$ state or the azimuthal phase gradient in the $m_l = \pm 1$ states).

Remark (Physical intuition and interpretation: why the total spin squared is $s(s+1)\hbar^2$ rather than $s^2\hbar^2$). When a spin-s particle is maximally polarized along the z axis, $S_z = s\hbar$, its total spin squared contains an additional contribution $+s\hbar^2$ beyond the classical-vector value $s^2\hbar^2$. Equation (7.3) gives a geometric and information-theoretic interpretation. The manifold of spin-polarization directions is the two-sphere S^2, with finite solid-angle area 4π. Freezing the spin vector exactly at the north pole without transverse fluctuations, $S_x = S_y = 0$, would require divergent angular Fisher information on the sphere. On the Hopf principal bundle $SU(2) \to S^2$, the Kähler information metric of Theorem 1 assigns $2s$ units of quantum Fisher information to each of the two orthogonal tangent directions (θ, φ) for a spin coherent state of order $n_s = 2s$. The total transverse trace is therefore $I^{(s)}_{\mathrm{QF},S^2} = 4s$. Multiplication by the elementary angular-momentum variance unit $(hj_0)^2 = \hbar^2/4$ yields the irreducible transverse zero-point angular-momentum fluctuation $\langle S_x^2 + S_y^2\rangle = s\hbar^2$.

Likewise, in an identical-fermion system the exchange phase shift $\pi\hbar$ enforces odd relative parity under $\boldsymbol{r}_{ab} \mapsto -\boldsymbol{r}_{ab}$, embedding the coincidence point $\boldsymbol{r}_{ab} = 0$ in a nodal plane ($m_l = 0$) or vortex line ($m_l = \pm 1$). For linear radial vanishing $\sqrt{\rho_{\uparrow\uparrow}} \propto r_{ab}$, this produces the divergent local radial Fisher kinetic cost $\hbar^2/(2\mu r_{ab}^2)$ in Eq. (7.4) alongside the $l = 1$ centrifugal barrier $\hbar^2/(\mu r_{ab}^2)$ (while deriving the macroscopic Fermi degeneracy equation of state of bulk matter requires evaluating the full N-fermion Slater-determinant kinetic energy functional).

Proof. **Step 1: Half-integer holonomy on the covering space $SU(2) \to SO(3)$ and calculation of spherical Kähler–Fisher curvature.** Since $SO(3) \cong SU(2)/\mathbb{Z}_2$, the closed 2π rotation loop $C_{2\pi}$ satisfies $2[C_{2\pi}] = 0$. Axiom I, Eq. (3.1), gives the holonomy $(k + 1/2)h = (2k+1)\pi\hbar$ in the sector $\chi = 1/2$. Write the dimensionless spin quantum number as $s = n_s/2$, where $n_s \in \mathbb{N}^+$ is the integer fiber weight of the Hopf principal bundle $SU(2) \to S^2$. The central element of $SU(2)$ acts on a section of order n_s by $(-1)^{n_s}$. Thus n_s is odd in the nontrivial holonomy sector and even in the trivial sector. The maximal longitudinal spin projection is $S_\parallel = n_s s_{\min} = s\hbar$, and its square is $S_\parallel^2 = (n_s h j_0)^2 = s^2\hbar^2$. By Theorem 1, Eq. (4.3), the dimensionless quantum-Fisher metric on the horizontal tangent space is $ds^2_{\mathrm{QFI}} = (2/\hbar)G_{\mathrm{FR}} = 4\int |\delta\psi_\perp|^2 d^d q$. Pulling ds^2_{QFI} back to the base manifold S^2 through an equivariant Hopf-bundle coherent section of fiber weight $n_s = 2s$ (for which $G_{\mathrm{FR}}|_{S^2} = s\hbar(d\theta^2 + \sin^2\theta\, d\varphi^2)$) induces the dimensionless spherical Kähler–Fisher metric tensor with conformal factor $n_s = 2s$: $F_{\theta\theta} = 2s$ and $F_{\varphi\varphi} = 2s\sin^2\theta$. Taking the trace with the inverse unit-sphere metric $g^{ab} = \mathrm{diag}(1, 1/\sin^2\theta)$ gives $I^{(s)}_{\mathrm{QF},S^2} = F_{\theta\theta} + F_{\varphi\varphi}/\sin^2\theta = 2s + 2s = 4s$. Substitution into Eq. (7.3) yields $\langle \boldsymbol{S}^2\rangle = s(s+1)\hbar^2$.

Step 2: Pauli information pressure and assumed spin–statistics compatibility. For a two-particle subsystem in center-of-mass and relative coordinates $\boldsymbol{R} = (\boldsymbol{q}_a + \boldsymbol{q}_b)/2$ and $\boldsymbol{r}_{ab} = \boldsymbol{q}_a - \boldsymbol{q}_b$, the gradient identity $|\nabla_a \ln\rho|^2 + |\nabla_b \ln\rho|^2 = \frac{1}{2}|\nabla_{\boldsymbol{R}} \ln\rho|^2 + 2|\nabla_{\boldsymbol{r}_{ab}} \ln\rho|^2$ multiplies the single-particle coefficient $(hj_0)^2/(2m)$ to give the relative-motion Fisher coefficient $(hj_0)^2/m = \hbar^2/(4m) = \hbar^2/(8\mu)$. In the fermionic sector of M_N, where $\Delta\Phi_{ab} = \pi\hbar$, the equal-spin relative amplitude vanishes linearly with odd parity: $\sqrt{\rho_{\uparrow\uparrow}} \propto r_{ab} \Rightarrow (\partial_{r_{ab}} \ln\rho_{\uparrow\uparrow})^2 \sim 4/r_{ab}^2$. Multiplication by $(hj_0)^2/m = \hbar^2/(8\mu)$ yields the local radial Fisher kinetic cost $\hbar^2/(2\mu r_{ab}^2)$ in Eq. (7.4), which combines with the $l = 1$ angular kinetic term $l(l+1)\hbar^2/(2\mu r_{ab}^2) = \hbar^2/(\mu r_{ab}^2)$ on S^2 (evaluated via $\int_{S^2} |\nabla_\Omega Y_{10}|^2 d\Omega = 2$ for $m_l = 0$ or azimuthal flow for $m_l = \pm 1$). On the joint orbital–frame quotient manifold M_N^{spin}, we impose compatibility of the holonomy assigned to the two-particle exchange path $[C_{\mathrm{exch}}]$ with that assigned to the single-particle frame's closed 2π rotation loop $[C_{2\pi}]$. Under this additional assumption

the line-bundle holonomy character satisfies $\Delta\Phi_{ab} \equiv \Delta\Phi_{2\pi} \pmod{h}$. The Finkelstein–Rubinstein field-configuration construction [123] and the relativistic quantum-field-theoretic spin–statistics theorem [124] provide relevant precedents; neither supplies a proof that these two loops are homotopic in the present point-particle configuration space. □

Corollary 9.1 (Information-geometric reconstruction and interpretation of three-dimensional hydrogen fine-structure splitting)**.** For an electron with dimensionless spin quantum number $s = 1/2$ and spin scale $S_{\parallel} = s\hbar = s_{\min} = hj_0 = \hbar/2$ in the Coulomb potential $V(r) = -e^2/(4\pi\epsilon_0 r)$, evaluate the three standard $O(\alpha_{\rm fs}^2)$ Foldy–Wouthuysen relativistic corrections [125, 126] on the stationary eigenstates of Theorem 6: the fourth-order relativistic kinetic-energy operator correction $\Delta H_{\rm rel} = -\langle\hat{p}^4\rangle/(8m^3c^2)$, the spin–orbit coupling from Thomas precession and spin-frame magnetic interaction in the Coulomb field, $\Delta H_{\rm SO} = [1/(2m^2c^2r)](dV/dr)\langle \boldsymbol{L}\cdot\boldsymbol{S}\rangle$, and the Darwin contact correction modeled as spatial resolution broadening of the Coulomb potential over an effective Compton smearing radius $r_{\rm smear} = (\sqrt{3}/2)\widetilde{\lambda}_C$ at the reduced Compton wavelength $\widetilde{\lambda}_C = \hbar/(mc)$, $\Delta H_{\rm D} = (\hbar^2/8m^2c^2)\langle\nabla^2 V\rangle = [e^2\hbar^2/(8m^2c^2\epsilon_0)]\rho(0)$. Their sum yields the exact $O(\alpha_{\rm fs}^2)$ hydrogen fine-structure spectrum for total angular-momentum quantum number $j_{\rm tot} = l \pm 1/2$:

$$E^{\rm FS}_{n,j_{\rm tot}} = E_n\left[1 + \frac{\alpha_{\rm fs}^2}{n^2}\left(\frac{n}{j_{\rm tot}+1/2} - \frac{3}{4}\right)\right], \tag{7.5}$$

where the fine-structure constant is $\alpha_{\rm fs} \equiv e^2/(4\pi\epsilon_0\hbar c) \simeq 1/137.036$.

Remark (Physical intuition and distinction between operator $\langle\hat{p}^4\rangle$ and first-order local osmotic moments)*.* In standard treatments of the Dirac equation, the three contributions to hydrogen fine structure (the mass–velocity term, the spin–orbit term, and the Darwin contact term) emerge together from the Foldy–Wouthuysen transformation [125, 126]. In particular, the Darwin term $\Delta H_{\rm D} \propto \rho(0)$ for s waves ($l = 0$) reflects Compton-scale spatial nonlocality (*Zitterbewegung*).

Within QICM, these three terms admit a transparent phase-space interpretation, while highlighting an important distinction between second-order and fourth-order momentum moments. At second order, Theorem 3 proves that the local first-gradient osmotic momentum $p_{\rm osm} = -(\hbar/2)\nabla\ln\rho$ reproduces the operator expectation $\langle\hat{p}^2\rangle = \int \rho(|\nabla\Phi|^2 + p_{\rm osm}^2)\, d^3q$ identically via integration by parts. At fourth order, however, expanding $\langle\hat{p}^4\rangle = \hbar^4\int|\nabla^2\psi|^2 d^3q = \int\rho[(|\nabla\Phi|^2 - \hbar^2\nabla^2\sqrt{\rho}/\sqrt{\rho})^2 + \hbar^2(\nabla^2\Phi + \nabla\ln\rho\cdot\nabla\Phi)^2]d^3q$ (where the second, phase-divergence term vanishes for stationary states with divergence-free current $\nabla\cdot(\rho\nabla\Phi) = 0$) shows that $\langle\hat{p}^4\rangle$ depends on the second-derivative Bohm quantum potential $-\hbar^2\nabla^2\sqrt{\rho}/(2m\sqrt{\rho})$ rather than the local first-gradient square $p_{\rm osm}^2/(2m)$ alone (for the hydrogen $1s$ ground state, $\int\rho\, p_{\rm osm}^4\, d^3q = (\hbar/a_0)^4$, whereas $\langle\hat{p}^4\rangle = 5(\hbar/a_0)^4$). Thus, reproducing the exact relativistic mass–velocity shift requires the full operator expectation $\langle\hat{p}^4\rangle$ (or second-derivative curvature terms), while the Darwin term $\Delta H_{\rm D} = (1/8)\widetilde{\lambda}_C^2\langle\nabla^2 V\rangle = [e^2\hbar^2/(8m^2c^2\epsilon_0)]\rho(0)$ corresponds to smoothing the Coulomb singularity $-e^2/(4\pi\epsilon_0 r)$ over a Compton sphere of calibrated root-mean-square radius $r_{\rm smear} = (\sqrt{3}/2)\widetilde{\lambda}_C$ using $\nabla^2(1/r) = -4\pi\delta^{(3)}(\boldsymbol{r})$.

Proof. **Step 1: Radial moments and the mass–velocity correction.** For the hydrogen eigenstates $|n, l, j_{\rm tot}\rangle$ of Theorem 6, use the Kramers–Pasternack radial expectation identities

$$\begin{aligned}\langle r^{-1}\rangle &= (n^2 a_0)^{-1},\\ \langle r^{-2}\rangle &= [n^3(l+1/2)a_0^2]^{-1},\\ \langle r^{-3}\rangle_{l>0} &= [n^3 l(l+1/2)(l+1)a_0^3]^{-1}.\end{aligned}$$

The mass–velocity relativistic kinetic-energy correction is

$$\langle H_{\rm rel}\rangle = -\frac{1}{2mc^2}\langle(E_n - V(r))^2\rangle = -\frac{\alpha_{\rm fs}^2|E_n|}{n^2}\left[\frac{n}{l+1/2} - \frac{3}{4}\right].$$

Step 2: Spin–orbit contribution for $l > 0$. For $l > 0$, Theorem 9, Eq. (7.3), gives $2\langle \boldsymbol{L}\cdot\boldsymbol{S}\rangle = [j_{\rm tot}(j_{\rm tot}+1) - l(l+1) - 3/4]\hbar^2$. For $j_{\rm tot} = l + 1/2$ and $j_{\rm tot} = l - 1/2$, multiply the respective spin–orbit factors by $\langle r^{-3}\rangle$ and add $\langle H_{\rm rel}\rangle$. Both cases reduce to Eq. (7.5).

Step 3: Darwin contact contribution for $l = 0$. For $l = 0$, hence $j_{\rm tot} = 1/2$, the spin–orbit term vanishes. The electron's intrinsic spatial uncertainty within the Compton scale $\widetilde{\lambda}_C$ produces the second-order Taylor broadening of the Coulomb potential

$$\delta V_{\rm D} = \frac{1}{6}\left(\frac{\sqrt{3}\,\widetilde{\lambda}_C}{2}\right)^2 \nabla^2 V = \frac{\hbar^2}{8m^2c^2}\nabla^2 V.$$

Substitution of $\rho_{n,0,0}(0) = (\pi n^3 a_0^3)^{-1}$ gives $\langle H_{\rm D}\rangle = \alpha_{\rm fs}^2|E_n|/n$. Adding $\langle H_{\rm rel}\rangle_{l=0} = -\alpha_{\rm fs}^2|E_n|(2n - 3/4)/n^2$ again yields Eq. (7.5) at $j_{\rm tot} = 1/2$. This result also explains the Dirac degeneracy of $2S_{1/2}$ and $2P_{1/2}$ and the 10.95 GHz fine-structure splitting $2P_{3/2} - 2P_{1/2}$. □

7.3 Theorem 10: Compton-wavelength finite-bandwidth ultraviolet effective regularization and calibration to the electron–positron rest-energy scale

Theorem 10 (Phenomenological Compton-wavelength DBI ultraviolet regularization and saturation properties)**.** A particle of mass $m > 0$, characterized by its rest-energy frequency $\nu_C = mc^2/h$ and the speed of light c, has spatial-resolution scales set by the Compton wavelength $\lambda_C \equiv h/(mc)$ and the reduced Compton wavelength $\widetilde{\lambda}_C \equiv 2j_0\lambda_C = \hbar/(mc)$. To regularize the short-wavelength ultraviolet behavior of the single-particle fluid approaching a vortex core or nodal surface ($|q - q_0| \lesssim \widetilde{\lambda}_C$), we construct the phenomenological Dirac–Born–Infeld (DBI) [127] ultraviolet effective energy functional

$$H_{\rm reg}[\rho,\Phi] = \int_{\mathbb{R}^d} \rho(q,t)\left[mc^2\sqrt{1 + \frac{|\nabla\Phi|^2}{m^2c^2} + I_{\rm reg}(\rho)} - mc^2 + V(q,t)\right] d^d q. \tag{7.6}$$

Here the local dimensionless amplitude-gradient regularization ratio is $I_{\rm reg}(\rho) \equiv \widetilde{\lambda}_C^2|\nabla\sqrt{\rho}|^2/[\rho + \xi_c\widetilde{\lambda}_C^2|\nabla\sqrt{\rho}|^2] = (p_{\rm osm}^2/m^2c^2)/[1 + \xi_c(p_{\rm osm}^2/m^2c^2)]$, where $\xi_c > 0$ is a dimensionless saturation parameter chosen phenomenologically as $\xi_c = 1/8$ to match the pair rest-energy scale $2mc^2$; this calibration does not derive a pair-production threshold or rate. Its formal Lorentz-covariant counterpart in four-dimensional Minkowski spacetime (including the rest-mass subtraction -1 matching Eq. (7.6)) is

$$A_{\rm DBI}^{\rm rel}[\varrho,\Phi] = -mc\int_{\mathbb{R}^{1,3}} \varrho(x) \times\left[\sqrt{1 - \frac{\eta^{\mu\nu}\partial_\mu\Phi\,\partial_\nu\Phi - m^2c^2}{m^2c^2} + I_{\rm reg}^{\rm cov}(\varrho)} - 1\right] d^4x,$$

where $I_{\rm reg}^{\rm cov}(\varrho)$ denotes the spatial-gradient invariant evaluated in the local rest frame of the timelike current J^μ (for a constant-amplitude plane wave with $I_{\rm reg}^{\rm cov} = 0$, stationary variation with respect to $\varrho(x)$ enforces $\sqrt{2 - \eta^{\mu\nu}\partial_\mu\Phi\,\partial_\nu\Phi/(m^2c^2)} - 1 = 0$, recovering the exact free relativistic Klein–Gordon mass shell $\eta^{\mu\nu}\partial_\mu\Phi\,\partial_\nu\Phi = m^2c^2$). This effective functional satisfies three exact properties (summarized in Fig. 3):

1. **Exact recovery of the quadratic functional in the low-energy regime.** When the characteristic spatial-gradient scale obeys $L \gg \widetilde{\lambda}_C$ and $|\nabla\Phi| \ll mc$, expanding $I_{\rm reg}(\rho) = (p_{\rm osm}^2/m^2c^2) - \xi_c(p_{\rm osm}^2/m^2c^2)^2 + \cdots$ with $p_{\rm osm}^2 = (hj_0)^2|\nabla\ln\rho|^2 \ll m^2c^2$ recovers the quadratic

energy functional of Axiom III, Eq. (3.6), at leading order, together with the complete fourth-order local gradient expansion:

$$H_{\rm reg}[\rho,\Phi] = \int_{\mathbb{R}^d} \rho \left[\frac{|\nabla\Phi|^2}{2m} + \frac{(hj_0)^2}{2m}|\nabla\ln\rho|^2 + V \right. \\ \left. -\frac{1}{8m^3c^2}\left(|\nabla\Phi|^2 + (hj_0)^2|\nabla\ln\rho|^2\right)^2 - \frac{\xi_c}{2m^3c^2}(hj_0)^4|\nabla\ln\rho|^4 + \cdots \right] d^dq. \tag{7.7}$$

(Because $I_{\rm reg}(\rho)$ is constructed from the local first-gradient osmotic momentum $p_{\rm osm} = -(\hbar/2)\nabla\ln\rho$ to regularize short-wavelength singularities at $|q-q_0| \lesssim \tilde{\lambda}_C$, its $O(c^{-2})$ fourth-gradient term $-\frac{1+4\xi_c}{8m^3c^2}\int \rho\, p_{\rm osm}^4\, d^dq$ differs from the second-derivative Foldy–Wouthuysen operator expectation $-\langle\hat{p}^4\rangle/(8m^3c^2)$ of Corollary 9.1 at atomic scales $a_0 = \alpha_{\rm fs}^{-1}\tilde{\lambda}_C \gg \tilde{\lambda}_C$.)

2. **Subluminal velocity saturation at a vortex-singularity core.** At the core $r_\perp \to 0$ of a coherent vortex with circulation quantum number n_l, the effective drift velocity $\boldsymbol{v}_{\rm reg} \equiv \delta H_{\rm reg}/\delta(\rho\nabla\Phi)$ is strictly bounded below c:

$$|\boldsymbol{v}_{\rm reg}(r_\perp)| \le \frac{c|n_l|\tilde{\lambda}_C}{\sqrt{r_\perp^2 + n_l^2\tilde{\lambda}_C^2}} < c \qquad (r_\perp > 0). \tag{7.8}$$

3. **Saturation of per-particle osmotic energy at a codimension-1 nodal hypersurface.** At any first-order codimension-1 nodal hypersurface $\rho(q_0) = 0$ (where the weighted Fisher density $\rho|\nabla\ln\rho|^2 = 4|\nabla\sqrt{\rho}|^2$ remains finite, while the unweighted per-particle osmotic momentum $p_{\rm osm} = -(\hbar/2)\nabla\ln\rho \propto |q-q_0|^{-1}$ diverges), the ratio $I_{\rm reg}(\rho)$ approaches the finite slope-independent upper limit $I_{\rm reg}^{\max} = 1/\xi_c$. With the phenomenological calibration $\xi_c = 1/8$ ($I_{\rm reg}^{\max} = 8$), the local per-particle osmotic energy factor saturates at the chosen electron–positron rest-energy scale $2mc^2$:

$$E_{\rm info}^{\max}|_{q=q_0} = mc^2\left(\sqrt{1+I_{\rm reg}^{\max}} - 1\right) \\ = mc^2\left(\sqrt{1+8} - 1\right) = 2mc^2. \tag{7.9}$$

Remark (Physical intuition and interpretation: phenomenological energy saturation near $\tilde{\lambda}_C$ and the need for a multiparticle completion). In the nonrelativistic Madelung fluid and Bohmian mechanics, the flow velocity $v = n_l\hbar/(mr_\perp) \to \infty$ exceeds c as the distance from a vortex core tends to zero, $r_\perp \to 0$. At a codimension-1 nodal hypersurface, $\rho \to 0$, the local osmotic momentum $p_{\rm osm} = -(\hbar/2)\nabla\ln\rho \to \infty$ also diverges. Theorem 10 attributes these ultraviolet divergences to the omission, in the nonrelativistic approximation, of the intrinsic Compton spatial-resolution limit $\tilde{\lambda}_C = \hbar/(mc)$ associated with rest mass.

The relativistic mass–energy dispersion relation $E = \sqrt{m^2c^4 + p^2c^2}$ is extended by the DBI-type functional (7.6) to include Fisher osmotic momentum. With the chosen saturation parameter, the local information energy at a node approaches the ceiling $2mc^2$, Eq. (7.9). *Interpreting localization below $\tilde{\lambda}_C$ as a motivation for a multiparticle completion is a heuristic of the present model; the identification of the ceiling with the pair rest-energy scale $2mc^2$ is imposed by calibration.* Schwinger's external-field calculation [128] obtains pair-creation probabilities from the imaginary part of the effective action and does not establish this localization-based saturation law. A transition to a particle–antiparticle production channel would require interacting field dynamics beyond the free-field functional of Theorem 8.

Schematic Comparison: Ultraviolet Behavior Near a Nodal Hypersurface ($q \to q_0$) and Vortex Core ($r_\perp \to 0$)

Physical Regime	**Nonrelativistic Quadratic Functional [Axiom III, Eq. (3.6)]**	**Compton-Scale Bounded-Bandwidth DBI Functional [Theorem 10, Eq. (7.6)]**
Infrared / Low-Energy Regime ($L \gg \widetilde{\lambda}_C$)	$T_{\rm flow} + T_{\rm info} = \int \rho\big[\frac{\lvert\nabla\Phi\rvert^2}{2m} + \frac{\hbar^2}{8m}\lvert\nabla \ln\rho\rvert^2\big] d^d q$; exact linear Schrödinger dynamics.	Recovers the quadratic functional up to $O(\widetilde{\lambda}_C^2/L^2)$ relativistic corrections [Eq. (7.7)], preserving low-energy linearity and no-signaling.
Vortex-Core Singularity ($r_\perp \to 0$)	Irrotational drift speed $v = n_l\hbar/(mr_\perp) \to \infty$ violates the relativistic light-speed bound for $r_\perp < \lvert n_l\rvert\widetilde{\lambda}_C$.	Effective drift speed saturates subluminally: $\lvert\boldsymbol{v}_{\rm reg}(r_\perp)\rvert \le c\lvert n_l\rvert\widetilde{\lambda}_C/\sqrt{r_\perp^2 + n_l^2\widetilde{\lambda}_C^2} < c$ for all $r_\perp > 0$ [Eq. (7.8)].
Codimension-1 Nodal Hypersurface ($q \to q_0$)	Osmotic momentum $p_{\rm osm} = -(\hbar/2)\nabla\ln\rho \propto \lvert q - q_0\rvert^{-1} \to \infty$; unweighted per-particle osmotic energy $p_{\rm osm}^2/(2m)$ diverges as $\lvert q-q_0\rvert^{-2} \to +\infty$ (while $\rho\lvert\nabla\ln\rho\rvert^2 = 4\lvert\nabla\sqrt{\rho}\rvert^2$ stays finite).	With the calibrated parameter $\xi_c = 1/8$, $I_{\rm reg}(\rho) \to 1/\xi_c = 8$ and the per-particle osmotic energy factor saturates at $E_{\rm info}^{\max} = mc^2(\sqrt{1+8} - 1) = 2mc^2$ [Eq. (7.9)], matching the e^-e^+ rest-energy scale by construction.

Figure 3: Quantitative comparison of ultraviolet behavior near a codimension-1 nodal hypersurface $q = q_0$ and vortex core $r_\perp = 0$. While the unweighted per-particle osmotic energy $p_{\rm osm}^2/(2m)$ and vortex drift speed $v = n_l\hbar/(mr_\perp)$ diverge as $\lvert q-q_0\rvert^{-2}$ and $r_\perp^{-1}$ in the nonrelativistic approximation, the phenomenological Compton-scale DBI regularization of Theorem 10 ensures subluminal vortex drift ($\lvert\boldsymbol{v}_{\rm reg}\rvert < c$) and saturates the per-particle nodal energy factor at $2mc^2$ ($\xi_c = 1/8$).

Proof. **Step 1: Covariant conservation, nonrelativistic reduction, and proof of the three properties in Theorem 10.** For the covariant action $A_{\rm DBI}^{\rm rel}[\varrho, \Phi]$, stationary variation with respect to $\Phi(x)$ yields the continuity equation $\partial_\mu J_{\rm DBI}^\mu = 0$ for the regularized four-current

$$J_{\rm DBI}^\mu(x) \equiv -\frac{\delta A_{\rm DBI}^{\rm rel}}{\delta(\partial_\mu\Phi(x))} = -\frac{\varrho(x)\,\eta^{\mu\nu}\partial_\nu\Phi(x)}{mc\sqrt{2 - \eta^{\alpha\beta}\partial_\alpha\Phi\,\partial_\beta\Phi/(m^2c^2) + I_{\rm reg}^{\rm cov}(\varrho)}}.$$

In the nonrelativistic reduction, setting $\Phi(x) = -mc^2t + \Phi_{\rm nr}(\boldsymbol{x}, t)$ with $\lvert\partial_t\Phi_{\rm nr}\rvert \ll mc^2$ and $A(\rho, \nabla\Phi_{\rm nr}) \equiv \lvert\nabla\Phi_{\rm nr}\rvert^2/(m^2c^2) + I_{\rm reg}(\rho)$ gives $2 - \eta^{\mu\nu}\partial_\mu\Phi\,\partial_\nu\Phi/(m^2c^2) = 1 + 2\partial_t\Phi_{\rm nr}/(mc^2) - (\partial_t\Phi_{\rm nr})^2/(m^2c^4) + \lvert\nabla\Phi_{\rm nr}\rvert^2/(m^2c^2)$. Expanding the Lagrangian density of $A_{\rm DBI}^{\rm rel}$ (with $d^4x = c\,dt\,d^3x$) to first order in $\partial_t\Phi_{\rm nr}/(mc^2)$ yields $-\varrho(\partial_t\Phi_{\rm nr})/\sqrt{1+A} - mc^2\varrho(\sqrt{1+A} - 1)$. In the low-energy regime $A \ll 1$ where $\sqrt{1+A} = 1 + O(c^{-2})$, the temporal term $-\varrho\,\partial_t\Phi_{\rm nr}$ integrates by parts over t to the canonical symplectic term $\int \Phi_{\rm nr}\partial_t\rho\,d^3x$, while in the single-particle phase-space formulation on $\mathcal{M}_{\rm ontic}$ the canonical symplectic structure $\Omega = \int \delta\rho \wedge \delta\Phi_{\rm nr}\,d^dx$ is retained directly with $H_{\rm reg}[\rho, \Phi_{\rm nr}]$ in Eq. (7.6) serving as the regularized Hamiltonian generator. Let $a \equiv \lvert\nabla\Phi\rvert^2/(m^2c^2)$ and $b \equiv \widetilde{\lambda}_C^2\lvert\nabla\sqrt{\rho}\rvert^2/\rho = (hj_0)^2\lvert\nabla\ln\rho\rvert^2/(m^2c^2)$. For $a, b \ll 1$, expanding $I_{\rm reg}(\rho) = b/(1+\xi_c b) = b - \xi_c b^2 + O(b^3)$ and $\sqrt{1+a+b-\xi_c b^2} - 1 = \frac{1}{2}(a+b) - \frac{1}{8}(a+b)^2 - \frac{1}{2}\xi_c b^2 + O((a+b)^3)$ yields Eq. (7.7). The variational derivative of $H_{\rm reg}$ with respect to $\rho\nabla\Phi$ gives the effective drift velocity

$$\boldsymbol{v}_{\rm reg} = \frac{\nabla\Phi/m}{\sqrt{1 + \lvert\nabla\Phi\rvert^2/(m^2c^2) + I_{\rm reg}(\rho)}}.$$

Substituting the vortex circulation $\lvert\nabla\Phi\rvert = mc\lvert n_l\rvert\widetilde{\lambda}_C/r_\perp$ and using $I_{\rm reg} \ge 0$ yields Eq. (7.8).

Near a codimension-1 nodal hypersurface at q_0, the signed real amplitude crosses zero linearly: $u(s) = a(q_0)s + O(s^2)$, where $a(q_0) \neq 0$. Thus $\rho(s) = a^2 s^2 + O(s^3)$, whereas $|\nabla\sqrt{\rho}|^2 = a^2 + O(s) > 0$ (so the weighted integrand $\rho I_{\mathrm{reg}}(\rho)$ vanishes smoothly at the node). Within $I_{\mathrm{reg}}(\rho)$, as $s \to 0$, the nonzero slope factor a^2 cancels between numerator and denominator, giving $\lim_{s\to 0} I_{\mathrm{reg}}(\rho) = 1/\xi_c = 8$ for the calibrated parameter $\xi_c = 1/8$. Substitution establishes Eq. (7.9).

Step 2: Scope of the phenomenological single-particle functional and ensemble no-signaling. Because Eq. (7.6) is a nonlinear functional of ρ, treating it as an exact fundamental evolution for entangled single-particle states would introduce nonlinear state dependence into reduced dynamics [91–93]. Within QICM, however, the fundamental microscopic dynamics at relativistic energies is the strictly quadratic multiparticle field-probability functional $H_{\mathrm{field}}[P, \Phi]$ of Theorem 8, Eq. (7.2), on the field configuration space $C = \{\varphi(\boldsymbol{x})\}$, which generates strictly linear functional Schrödinger evolution and preserves CPTP ensemble no-signaling across all energy scales. Equation (7.6) serves as a phenomenological single-particle ultraviolet regularization that models kinematic velocity and osmotic-energy saturation at the calibrated pair rest-energy scale $2mc^2$; deriving pair-production dynamics or an exact effective action from interacting QED remains an open frontier noted in Section 8.4. □

7.4 Theorem 11: The Fisher-information Itô drift identity, no-signaling under general local operations, and the monitored boundary-channel capacity bound

Theorem 11 (Second-order Itô drift of Fisher information, ensemble no-signaling under general local operations, and the monitored boundary-channel capacity bound)**.**

1. **Exact second-order Itô drift of Fisher information and channel backaction heating power.** Under the stochastic evolution equation of Axiom III, Eq. (3.8a), on any localized stopping-time domain $t < \tau_\varepsilon$ with bounded measurement kernel (Theorem 7), the pathwise second-order Itô drift of $I_{\mathrm{F}}[\rho] = \int (|\nabla\rho|^2/\rho)\, d^d q$ is independent of $\nabla\rho$ and equals $[2\gamma_{\mathrm{info}}(t)/\Delta_M^2(t)](\int_{\mathbb{R}^d} \rho(q,t)|\nabla M(q)|^2 d^d q)\, dt$. Taking the unconditional expectation fixes the mean Fisher drift and the average backaction power injected by the channel in a pure configuration measurement ($d\Phi_{\mathrm{back}} \equiv 0$):

$$\begin{aligned} d\mathbb{E}\big[I_{\mathrm{F}}[\rho]\big]_{\mathrm{meas}} &= \mathbb{E}\left[\frac{2\gamma_{\mathrm{info}}(t)}{\Delta_M^2(t)} \int_{\mathbb{R}^d} \rho(q,t)|\nabla M(q)|^2\, d^d q\right] dt, \\ \frac{dE_{\mathrm{channel}}}{dt} &= \frac{\hbar^2 \kappa_{\mathrm{meas}}(t)}{2m}\, \mathbb{E}\left[\int_{\mathbb{R}^d} \rho(q,t)|\nabla M(q)|^2\, d^d q\right]. \end{aligned} \tag{7.10}$$

2. **Ensemble no-superluminal-signaling under general local operations.** Let spacelike-separated systems A and B be in an arbitrary entangled state (ρ_{AB}, Φ_{AB}), with no action-at-a-distance interaction potential, $V_{AB} = V_A(q_A) + V_B(q_B)$. When A undergoes a local configuration measurement $M_A(q_A)$ with deterministic bare coupling rate $\kappa_{\mathrm{meas}}^{(A)}(t)$, the partial Fisher information injected into B by the measurement on A is identically zero, $d\mathbb{E}[I_{\mathrm{F},B}[\rho_{AB}]]_{\mathrm{meas}} \equiv 0$. More generally, when A undergoes any local measurement or arbitrary local CPTP quantum operation, regardless of the simultaneous local Hamiltonian evolution or local measurement channels $\{\kappa_b, M_b(q_B)\}$ on B, the nonselective reduced coherence kernel of B obeys, for every $t \geq 0$, an autonomous linear evolution entirely independent of the parameters, measurement-basis choice,

and local CPTP operations on A:

$$\begin{aligned}\frac{\partial}{\partial t}\mathbb{E}[\varrho_B^{(\omega)}(q_B,q_B';t)] &= -\frac{i}{\hbar}[\hat{H}_B,\mathbb{E}[\varrho_B^{(\omega)}(t)]](q_B,q_B') \\ &\quad + L_B\mathbb{E}[\varrho_B^{(\omega)}(t)] \\ \Longrightarrow \qquad \frac{\partial}{\partial\kappa_{\mathrm{meas}}^{(A)}}\mathbb{E}[\rho_B(q_B,t)] &\equiv 0 \qquad (\forall t\geq 0).\end{aligned} \tag{7.11}$$

3. **Monitored boundary-channel capacity bound across spatial bipartitions.** For an N-body lattice or continuum system partitioned into a spatial subregion A and its complement $\bar{A}$, suppose information exchange across the boundary ∂A is mediated by $N_{\mathrm{bonds}}(\partial A)$ local measurement or communication channels, each constrained by Axiom III with pure entanglement interaction energy standard deviation at most ΔE_{bond} (allowing bidirectional readout across each bond). Then the rate of classical Shannon mutual-information extraction $I_{\mathrm{ch}}(A:\bar{A})$ across ∂A obeys the monitored boundary-channel capacity bound:

$$\begin{aligned}\frac{dI_{\mathrm{ch}}(A:\bar{A})}{dt} &\leq \frac{2N_{\mathrm{bonds}}(\partial A)\Delta E_{\mathrm{bond}}}{h} \quad \mathrm{nat/s}, \\ \Delta K_{\max}(\partial A,t) &= \left\lfloor\frac{\Delta I_{\mathrm{ch}}(A:\bar{A})}{j}\right\rfloor.\end{aligned} \tag{7.12}$$

Remark (Physical intuition and interpretation: measurement backaction work and the monitored boundary-channel capacity bound). A position measurement necessarily increases momentum fluctuations, producing measurement backaction heating, whereas a QND measurement of discrete energy levels does not produce this type of backaction. Equation (7.10) gives a quantitative explanation. Stochastic updating through the measurement channel compresses the wavepacket width and injects spatial Fisher information at rate $d\mathbb{E}[I_{\mathrm{F}}[\rho]]_{\mathrm{meas}}/dt = 4\kappa_{\mathrm{meas}}\int\rho|\nabla M(q)|^2\,d^dq$. For continuous position measurement, $M(q)=q$ and $|\nabla M|=1$, the injected Fisher information is converted directly into increased momentum variance and kinetic energy, $dE_{\mathrm{channel}}/dt = \hbar^2\kappa_{\mathrm{meas}}/(2m)$. For an operator that is constant within each discrete support, $\nabla M = 0$ in each support, the internal local gradients remain unchanged and backaction heating is zero.

In a composite system AB, a local measurement operator on A satisfies $\nabla_{q_B}M_A(q_A)\equiv 0$. Thus *measurement on A injects no partial Fisher information or energy into the spacelike-separated system B*. In a many-body lattice system, monitored information exchange between a subregion A and its exterior $\bar{A}$ occurs through the $N_{\mathrm{bonds}}(\partial A)$ local coupling bonds crossing ∂A. Under Axiom III, each directional channel across a bond has capacity at most $\Delta E_{\mathrm{bond}}/h$, yielding the factor of 2 for bidirectional exchange in Eq. (7.12). Note that Eq. (7.12) governs *monitored channel mutual information* $I_{\mathrm{ch}}(A:\bar{A})$ under Axiom III (Theorem 7, Eq. (5.34)), whereas coherent unitary von Neumann entanglement entropy generation $dS_{\mathrm{vN}}(\hat{\rho}_A)/dt$ in a closed bipartite system is governed by the Small Incremental Entangling (SIE) bound $dS_{\mathrm{vN}}(\hat{\rho}_A)/dt \leq c_{\mathrm{SIE}}\|\hat{H}_{\mathrm{bond}}\|\ln(\min\{d_A,d_{\bar{A}}\})/\hbar$ [129] (for instance, two qubits evolving unitarily under $\hat{H}=\hbar g\,\hat{\sigma}_x\otimes\hat{\sigma}_x$ from $|00\rangle$ reach a maximally entangled Bell state with quantum mutual information $I_{\mathrm{vN}}(A:B)=2\ln 2\approx 1.386\,\mathrm{nat}$ at $T=\pi/(4g)$, whereas a monitored channel governed by Eq. (7.12) with $\Delta E_{\mathrm{bond}}=\hbar g$ extracts at most $2\Delta E_{\mathrm{bond}}T/h = 1/4\,\mathrm{nat}$ in the same interval).

Proof. **Step 1: Second-order Itô drift of Fisher information and channel heating power.** Write the one-step stochastic density increment as $\delta\rho=\epsilon_t\widetilde{M}_t\rho$, where $\epsilon_t=\sqrt{2\gamma_{\mathrm{info}}(t)}\,dW_t$ and $\widetilde{M}_t(q)=(M(q)-\bar{M}_t)/\Delta_M(t)$. Expand the Fisher-information integrand $|\nabla(\rho+\delta\rho)|^2/(\rho+\delta\rho)$

directly. The identity $\nabla(\rho+\delta\rho) = (1+\epsilon_t\widetilde{M}_t)\nabla\rho + \epsilon_t\rho\nabla\widetilde{M}_t$ gives an exact match to the denominator factor $(1+\epsilon_t\widetilde{M}_t)$:

$$\frac{|\nabla(\rho+\delta\rho)|^2}{\rho+\delta\rho} = (1+\epsilon_t\widetilde{M}_t)\frac{|\nabla\rho|^2}{\rho} + 2\epsilon_t\nabla\rho\cdot\nabla\widetilde{M}_t + \frac{\epsilon_t^2}{1+\epsilon_t\widetilde{M}_t}\rho|\nabla\widetilde{M}_t|^2. \tag{7.13}$$

The first two terms on the right-hand side of Eq. (7.13) are strictly linear in the stochastic increment ϵ_t. Their second derivatives with respect to ϵ_t vanish, and their first-order Itô expectations are zero. The positive and negative stretching of the pre-existing density gradient in a single stochastic update cancel in the first-order average. Only the third term describes the net wavepacket compression caused by the measurement kernel's own spatial gradient $\nabla\widetilde{M}_t$. Expanding $(1+\epsilon_t\widetilde{M}_t)^{-1} = 1-\epsilon_t\widetilde{M}_t + O(\epsilon_t^2)$ in the third term on the stopped domain $t < \tau_\varepsilon$ shows that its quadratic Itô variation $\epsilon_t^2 = 2\gamma_{\rm info}(t)dt$ (with higher-order terms of order $O(dt^{3/2})$ vanishing in the continuous-time Itô limit) continuously injects positive spatial Fisher information and backaction kinetic energy into the system. Substitute $\nabla\widetilde{M}_t(q) = \nabla M(q)/\Delta_M(t)$ and integrate over all space. Combining $(hj_0)^2/(2m) = \hbar^2/(8m)$ with $\gamma_{\rm info} = 2\kappa_{\rm meas}\Delta_M^2$ establishes Eq. (7.10).

Step 2: Ensemble no-superluminal-signaling under general local operations. For measurement of $M_A(q_A)$ on subsystem A, the identity $\nabla_{q_B}M_A(q_A) \equiv 0$, together with Eq. (7.10), gives $d\mathbb{E}[I_{{\rm F},B}[\rho_{AB}]]_{\rm meas} \equiv 0$. When local measurement channels act simultaneously on A and B, item 4 of Theorem 7, Eq. (5.33), implies that the joint nonselective density kernel $\varrho_{AB} = \mathbb{E}[\psi_{AB}\psi_{AB}^*]$ obeys the state-independent linear Lindblad master equation

$$\partial_t\varrho_{AB} = -\frac{i}{\hbar}[\hat{H}_A+\hat{H}_B,\varrho_{AB}] + (L_A\otimes I_B + I_A\otimes L_B)\varrho_{AB}.$$

Take the partial trace by setting $q_A = q_A'$ and integrating, or use cyclicity of the partial trace for any local CPTP Kraus map. The L_A term vanishes identically at $q_A = q_A'$. Since L_B is a strictly linear superoperator, different pure-state ensemble decompositions have the same marginal evolution. This proves Eq. (7.11), the Simon–Bužek–Gisin necessary-and-sufficient no-signaling criterion [92].

Finally, summing the Axiom III elementary-channel capacity bound (5.34) over the $N_{\rm bonds}(\partial A)$ local boundary channels (counting both $A\to\bar{A}$ and $\bar{A}\to A$ readout directions per bond) yields the monitored boundary-channel capacity bound (7.12). This monitored-channel result follows from Axiom III. Related closed-system results include finite propagation bounds [130, 131] and bounds on Hamiltonian entanglement-generation rates [129, 131]; the ground-state area law for gapped one-dimensional systems [132] and equilibrium mutual-information area laws [133] concern distinct settings and do not supply a proof of the present capacity bound. □

8 Testable experimental predictions, synthesis, and scope

8.1 Prediction I: Markovian capacity bounds on continuous compression and discrete-branch readout across platforms

Within the baseline Markovian continuous-measurement framework of Axiom III and Theorem 7, the capacity ceiling $C_{\max} = \Delta E_{\rm int}/h$ constrains two distinct operational tasks: (a) continuous-variable wave-packet compression and (b) discrete-branch state discrimination.

First, for a continuous-spectrum observable $M(q)$ with initial objective uncertainty $\Delta_{\rm init} = \Delta_M(0)$ and target instrumental resolution $\Delta_{\rm det}$, define the resolution-compression ratio by $\eta \equiv \Delta_{\rm init}/\Delta_{\rm det} > 1$. On a measurement-invariant family with $\mu_3(t) \equiv 0$, such as a Gaussian family when Hamiltonian

transport contributes nothing to the first two measured moments, the first relation in Theorem 7, Eq. (5.28b), holds exactly (under H2b alone it represents the leading measurement-dominated approximation):

$$\begin{aligned} \Delta^2_{\rm det} &= \Delta^2_{\rm init}\exp\left(-2\int_0^{\tau_{\rm collapse}}\gamma_{\rm info}(t)\,dt\right) \\ \Longrightarrow \qquad \Delta I_{\rm req} &= \int_0^{\tau_{\rm collapse}}\gamma_{\rm info}(t)\,dt = \ln\left(\frac{\Delta_{\rm init}}{\Delta_{\rm det}}\right)\ \text{nat}. \end{aligned} \tag{8.1}$$

At every time t along the trajectory, the Nyquist–Planck capacity bound of Axiom III, Eq. (3.7), requires $\gamma_{\rm info}(t) \le C_{\max} = \Delta E_{\rm int}/h$. If $\gamma_{\rm info}(t)$ is maintained near the capacity ceiling $C_{\max}$ via adaptive coupling $\kappa_{\rm meas}(t) = \gamma_{\rm info}/[2\Delta^2_M(t)]$, the compression time obeys the logarithmic lower bound

$$\begin{aligned} \tau_{\rm collapse} \ge \tau_{\rm Nyq}(\eta) &\equiv \frac{\Delta I_{\rm req}}{C_{\max}} \\ &= \frac{h}{\Delta E_{\rm int}}\ln\left(\frac{\Delta_{\rm init}}{\Delta_{\rm det}}\right) = \frac{2\pi\hbar}{\Delta E_{\rm int}}\ln\left(\frac{\Delta_{\rm init}}{\Delta_{\rm det}}\right). \end{aligned} \tag{8.2}$$

Conversely, if the bare coupling rate $\kappa_{\rm meas}$ is held constant without adaptive feedback (preserving the state-independent linear Lindblad ensemble evolution (5.33)) and satisfies the initial capacity constraint $2\kappa_{\rm meas}\Delta^2_{\rm init} \le C_{\max}$, the algebraic variance solution $\Delta^2_M(t) = \Delta^2_{\rm init}/(1+4\kappa_{\rm meas}\Delta^2_{\rm init}t)$ of Theorem 7 yields the steeper algebraic compression-time bound $\tau_{\rm collapse} \ge (\eta^2-1)/(2C_{\max}) > \tau_{\rm Nyq}(\eta)$.

Second, for a finite set of N discrete QND branches (such as binary qubit readout, $N=2$), Theorem 7, Eq. (5.34), gives the exact ensemble Shannon mutual-information identity $I_{\rm Sh}(K;\mathcal{F}_T) = \mathbb{E}[\int_0^T \gamma_{\rm info}(t)\,dt] \le C_{\max}T$ at a deterministic observation horizon T. For a symmetric binary prior $P_1(0) = P_2(0) = 1/2$, achieving an assignment fidelity $F = 1-\varepsilon_{\rm err}$ requires extracting discrete Shannon mutual information $\Delta I_{\rm Sh}(F) = \ln 2 - H_b(\varepsilon_{\rm err}) \le \ln 2 \approx 0.6931$ nat, which imposes the ensemble horizon bound $T \ge \Delta I_{\rm Sh}(F)/C_{\max}$. We emphasize the operational distinction between these two benchmarks: $\Delta I_{\rm Sh}(F) \le \ln 2$ bounds the discrete Shannon information of the binary branch label $K \in \{+1,-1\}$, whereas $\Delta I_{\rm req} = \ln\eta$ in Eq. (8.2) measures the continuous phase-space compression (or effective homodyne signal-to-noise ratio SNR $= \eta$) of a Gaussian pointer distribution [20].

Complete three-component covariance dynamics for non-QND continuous position measurements with external-potential curvature. For a non-QND continuous position measurement $M(q) = q$ in an external potential of local curvature $k_{\rm eff} \equiv \langle\partial_q^2 V\rangle = m\omega_0^2$ with extracted information rate $\gamma_{\rm info} = 2\eta_{\rm det}\kappa_{\rm meas}V$ (at detector collection efficiency $0 < \eta_{\rm det} \le 1$, reducing to $\gamma_{\rm info} = 2\kappa_{\rm meas}V$ at $\eta_{\rm det} = 1$), a conditional Gaussian packet with second central moments $(V, C, P) \equiv (\Delta_q^2, \sigma_{qp}, \Delta_p^2)$ (including any mixed-state branch-momentum dispersion from Section 2.3) has linear phase gradient $\partial_q\Phi = \bar{p}_t + (C/V)(q-\bar{q}_t)$. Under Eqs. (3.8a)–(3.8b) with $d\Phi_{\rm back} = 0$, its conditional centroid $(\bar{q}_t, \bar{p}_t)$ obeys the coupled Itô equations $d\bar{q}_t = (\bar{p}_t/m)dt + \sqrt{2\gamma_{\rm info}V}\,dW_t$ and $d\bar{p}_t = -\langle\partial_q V\rangle\,dt + \sqrt{2\gamma_{\rm info}/V}\,C\,dW_t$. Subtracting the second-order Itô drift corrections $(d\bar{q}_t)^2 = 2\gamma_{\rm info}V\,dt$, $d\bar{q}_t\,d\bar{p}_t = 2\gamma_{\rm info}C\,dt$, and $(d\bar{p}_t)^2 = (2\gamma_{\rm info}C^2/V)dt$ from the raw second moments and adding the Fisher backaction heating $dE_{\rm channel} = [\hbar^2\kappa_{\rm meas}/(2m)]dt = [\gamma_{\rm info}\hbar^2/(4m\eta_{\rm det}V)]dt$ of Theorem 11 [Eq. (7.10)] yields the closed Kalman–Bucy/Belavkin Riccati equations (shown in Eq. (8.3) for $\eta_{\rm det} = 1$, while for $0 < \eta_{\rm det} < 1$ the momentum-diffusion term $\gamma_{\rm info}\hbar^2/(2V)$ is replaced

by $\gamma_{\rm info}\hbar^2/(2\eta_{\rm det}V)$):

$$\begin{aligned}\dot{V} &= \frac{2C}{m} - 2\gamma_{\rm info}V,\\ \dot{C} &= \frac{P}{m} - k_{\rm eff}V - 2\gamma_{\rm info}C,\\ \dot{P} &= -2k_{\rm eff}C + \frac{\gamma_{\rm info}\hbar^2}{2V} - \frac{2\gamma_{\rm info}C^2}{V}.\end{aligned} \tag{8.3}$$

In the ideal limit $\eta_{\rm det}=1$, direct differentiation of the symplectic-area determinant $D_\Sigma^2 = VP - C^2$ along Eq. (8.3) yields the exact identity $(d/dt)(VP-C^2) = 2\gamma_{\rm info}[\hbar^2/4 - (VP-C^2)]$ (whereas for $\eta_{\rm det}<1$, D_Σ^2 relaxes toward the mixed stationary manifold $VP - C^2 = \hbar^2/(4\eta_{\rm det})$). Consequently, at $\eta_{\rm det}=1$, any mixed Gaussian state with $VP - C^2 > \hbar^2/4$ purifies exponentially toward the minimum-uncertainty manifold $VP - C^2 = \hbar^2/4$, on which the symplectic area is strictly conserved, $(d/dt)(VP-C^2)=0$. In the free-particle limit, $k_{\rm eff}=0$, and for constant $\gamma_{\rm info}$, the system approaches the dynamically balanced minimum-uncertainty stationary state $V_\infty = \hbar/(2m\gamma_{\rm info})$, $C_\infty = \hbar/2$, and $P_\infty = m\gamma_{\rm info}\hbar$. In the strong-measurement regime, $N_{\rm SM} \equiv \Delta_{\rm det}^2/V_\infty \gg 1$, neglecting the transport term $2C/m$ throughout the compression interval recovers the logarithmic scaling in Eq. (8.2). Under weak continuous monitoring, $N_{\rm SM} \sim 1$, the packet width approaches a stationary balance determined by mass and external potential.

Pure entanglement interaction energy standard deviation and illustrative benchmarks across platforms.

1. **Energy standard deviation after subtraction of local marginal shifts.** Under the gauge-invariant definition in Section 2.2, subtracting the marginal partial traces on the system S and probe E gives the pure entanglement interaction operator and its exact standard deviation:

$$\begin{aligned}\hat{H}_{\rm int}^{\rm ent}(t) \equiv \hat{H}_{\rm int} &- {\rm Tr}_E(\hat{H}_{\rm int}\hat{\rho}_E)\otimes\hat{I}_E\\ &- \hat{I}_S\otimes{\rm Tr}_S(\hat{H}_{\rm int}\hat{\rho}_S) + \langle\hat{H}_{\rm int}\rangle\hat{I}_{SE},\\ \Delta E_{\rm int}(t) \equiv \sqrt{\left\langle\left(\hat{H}_{\rm int}^{\rm ent}(t)\right)^2\right\rangle_t - \left\langle\hat{H}_{\rm int}^{\rm ent}(t)\right\rangle_t^2}&.\end{aligned} \tag{8.4}$$

   To evaluate the channel capacity $C_{\max} = \Delta E_{\rm int}/h$ as an intrinsic apparatus benchmark prior to collapse, $\Delta E_{\rm int}$ is evaluated on the initial unentangled superposition state $\hat{\rho}_S(0)\otimes\hat{\rho}_E(0)$ (where $\langle\hat{H}_{\rm int}^{\rm ent}(0)\rangle = 0$ identically).

2. **Illustrative capacity benchmark and experimental context for dispersive readout in superconducting circuit QED.** For dispersive readout of a superconducting transmon qubit coupled to a microwave resonator, as in Murch et al. (2013) [134] and Walter et al. (2017) [135] (see also Clerk et al. [20]), the dispersive interaction is $\hat{H}_{\rm int} = \hbar\chi_{\rm tot}\hat{a}^\dagger\hat{a}\hat{\sigma}_z$. For an initial equatorial qubit superposition ($\langle\hat{\sigma}_z\rangle = 0$, $\Delta\sigma_z = 1$) and a coherent intracavity pointer state with mean photon number $\bar{n}$ ($\Delta n = \sqrt{\bar{n}}$), Eq. (8.4) gives $\hat{H}_{\rm int}^{\rm ent} = \hbar\chi_{\rm tot}(\hat{n}-\bar{n})\hat{\sigma}_z$ and $\Delta E_{\rm int} = \hbar|\chi_{\rm tot}|\sqrt{\bar{n}}$. Adopting an illustrative parameter baseline of $|\chi_{\rm tot}|/(2\pi) = 10\,{\rm MHz}$ and $\bar{n}=9$ ($\sqrt{\bar{n}}=3$) yields a maximum channel capacity $C_{\max} = \Delta E_{\rm int}/h = 3.0\times10^7\,{\rm nat/s}$ and a single-step resolution time scale $\Delta t_{\min}(j) = j/C_{\max} = 2.65\,{\rm ns}$ (serving as a theoretical reference point alongside experimental studies of superconducting quantum trajectories [136], variable-strength backaction [137], and monitored quantum jumps [138]). Evaluating Eq. (8.2) at two illustrative benchmarks gives: (i) for a 10-fold continuous Gaussian pointer compression benchmark $\eta = \Delta_{\rm init}/\Delta_{\rm det} = 10$ ($\Delta I_{\rm req} = \ln 10 \approx 2.303\,{\rm nat}$), Eq. (8.2) gives

$$\tau_{\rm Nyq}(\eta=10) = \frac{\ln 10}{3.0\times10^7\,{\rm s}^{-1}} \approx 76.8\,{\rm ns} < \tau_{\rm exp}(99.2\%) = 88\,{\rm ns}, \tag{8.5}$$

where the 88 ns readout window of Walter et al. [135] is juxtaposed for order-of-magnitude context rather than as a matched-parameter test: their experiment operated at $\chi/(2\pi) = -7.9\,\mathrm{MHz}$ and $n_{\mathrm{drive}} = 2.5 \pm 0.25$, reporting discriminative fidelities $F_{\mathrm{W}} = 1 - P(e|g) - P(g|e)$ of 98.25% in 48 ns and 99.2% in 88 ns (corresponding to equal-prior correct-assignment probabilities of 99.125% and 99.6%, with T_1 state transitions contributing alongside Gaussian readout noise). (ii) Similarly, in an idealized Gaussian homodyne model where a target single-shot error $\varepsilon_{\mathrm{err}} = 1 - F$ maps to SNR $= \eta$ via $\varepsilon_{\mathrm{err}} = \frac{1}{2}\,\mathrm{erfc}(\mathrm{SNR}/2\sqrt{2})$, setting $F = 98.25\%$ directly ($\varepsilon_{\mathrm{err}} = 1.75\%$) gives SNR ≈ 4.22, $\tau_{\mathrm{Nyq}}(\eta = 4.22) = \ln(4.22)/C_{\max} \approx 48.0\,\mathrm{ns}$, and a discrete binary Shannon horizon $\Delta I_{\mathrm{Sh}}/C_{\max} \approx 20.2\,\mathrm{ns}$, whereas using the symmetric single-branch assignment probability $F_{\mathrm{assign}} = (1 + F_{\mathrm{W}})/2 = 99.125\%$ ($\varepsilon_{\mathrm{err}} = 0.875\%$) gives SNR ≈ 4.75, $\tau_{\mathrm{Nyq}}(\eta = 4.75) \approx 52.0\,\mathrm{ns}$, and $\Delta I_{\mathrm{Sh}}/C_{\max} \approx 21.4\,\mathrm{ns}$. Table 5 extends these order-of-magnitude juxtapositions to cavity optomechanics [139, 140] and trapped-ion fluorescence readout [141], explicitly distinguishing QICM's illustrative capacity parameterizations (which in the optomechanical and trapped-ion rows use heuristic rate mappings $g_{\mathrm{eff}}/(2\pi)$ and Γ_{det} rather than a microscopic evaluation of Eq. (8.4)) from the experimental metrics reported in Refs. [140, 141]. When $\Delta E_{\mathrm{int}}(t)$ varies in time along a measurement, the general integrated form of Axiom III is $\Delta I_{\mathrm{ex}}(T) \le h^{-1} \int_0^T \mathbb{E}[\Delta E_{\mathrm{int}}(t)]\, dt$.

3. **Prospective matched-parameter experimental test discriminating Axiom III from unconstrained Markovian SME in dispersive cQED.** Because the literature comparisons in Table 5 do not use matched parameters on a single device, we formulate a direct, prospective falsifiability test on a single superconducting circuit-QED readout apparatus. For a transmon qubit prepared in an equatorial superposition ($\Delta\sigma_z(0) = 1$) dispersively coupled via $\hat{H}_{\mathrm{int}} = \hbar\chi_{\mathrm{tot}}\hat{a}^\dagger\hat{a}\hat{\sigma}_z$ to a readout resonator of linewidth κ_{cav}, quantum efficiency η_{det}, and steady-state intracavity coherent photon number $\bar{n}$, standard cavity-eliminated Markovian SME theory [19, 20] predicts an initial information-extraction rate scaling *linearly* with $\bar{n}$:

$$\gamma_{\mathrm{info}}^{\mathrm{SME}}(\bar{n}) = 2\kappa_{\mathrm{meas}}^{\mathrm{SME}}\Delta_{\sigma_z}^2(0) = \frac{8\eta_{\mathrm{det}}\chi_{\mathrm{tot}}^2\bar{n}}{\kappa_{\mathrm{cav}}\left[1 + (2\chi_{\mathrm{tot}}/\kappa_{\mathrm{cav}})^2\right]}, \qquad C_{\max}(\bar{n}) = \frac{\Delta E_{\mathrm{int}}(0)}{h} = \frac{|\chi_{\mathrm{tot}}|\sqrt{\bar{n}}}{2\pi}.$$

Whereas $\gamma_{\mathrm{info}}^{\mathrm{SME}}(\bar{n}) \propto \bar{n}$, the pure entanglement capacity ceiling $C_{\max}(\bar{n})$ of Eq. (3.7) scales sublinearly as $\sqrt{\bar{n}}$. The two curves cross at the critical intracavity photon number

$$\bar{n}_{\mathrm{crit}} = \left(\frac{\kappa_{\mathrm{cav}}\left[1 + (2\chi_{\mathrm{tot}}/\kappa_{\mathrm{cav}})^2\right]}{16\pi\,\eta_{\mathrm{det}}|\chi_{\mathrm{tot}}|}\right)^2$$

(for example, at $|\chi_{\mathrm{tot}}|/\kappa_{\mathrm{cav}} = 0.25$ and $\eta_{\mathrm{det}} = 0.60$, $\bar{n}_{\mathrm{crit}} \approx 0.028$ photons, whereas at $|\chi_{\mathrm{tot}}|/(2\pi) = 1\,\mathrm{MHz}$, $\kappa_{\mathrm{cav}}/(2\pi) = 40\,\mathrm{MHz}$, and $\eta_{\mathrm{det}} = 0.50$, $\bar{n}_{\mathrm{crit}} \approx 2.54$ photons). Sweeping $\bar{n}$ across $[0.2\,\bar{n}_{\mathrm{crit}},\, 10\,\bar{n}_{\mathrm{crit}}]$ on the same device after cavity ring-up ($t \gg \kappa_{\mathrm{cav}}^{-1}$) while independently calibrating χ_{tot}, κ_{cav}, η_{det}, and $\bar{n}$ via AC Stark shift and photon-number calibration provides a parameter-free test: observing linear growth $\gamma_{\mathrm{info}}(\bar{n}) > |\chi_{\mathrm{tot}}|\sqrt{\bar{n}}/(2\pi)$ for $\bar{n} > \bar{n}_{\mathrm{crit}}$ would falsify the direct qubit–resonator capacity identification in Eq. (3.7) (demonstrating that the resonator acts as a coherent transducer and that the capacity bound applies instead to the resonator–transmission-line radiative coupling with bandwidth $\kappa_{\mathrm{cav}}\bar{n}$), whereas observing sublinear saturation at $C_{\max}(\bar{n}) \propto \sqrt{\bar{n}}$ would confirm that the internal qubit–pointer interaction $\hat{H}_{\mathrm{int}}$ directly bottlenecks information extraction.

Table 5: Illustrative assumptions for the channel capacity $C_{\max}$, single-quantum time scale $\Delta t_{\min}(j)$, and conditional compression benchmark τ_{Nyq}, alongside separately reported experimental observations. These comparisons do not calibrate or test the proposed capacity bound.

Experimental platform	Illustrative coupling parameters and probe state	Assumed capacity $C_{\max}$	Model resolution scale $\Delta t_{\min}(j)$	Model τ_{Nyq} benchmark and reported observations
Superconducting cQED dispersive readout [135]	Illustrative $\lvert\chi_{\mathrm{tot}}\rvert/(2\pi) = 10\,\mathrm{MHz}$; $\bar{n} = 9$	3.00×10^{7} nat/s (model)	2.65 ns	Assumed $\eta = 10$: **76.8 ns**; $\eta = 4.22$ (4.75): **48.0 ns** (**52.0 ns**). Reported: **88 ns** ($F_{\mathrm{W}} = 99.2\%$); **48 ns** ($F_{\mathrm{W}} = 98.25\%$).
Cavity-optomechanical membrane under continuous position monitoring [139, 140]	Assumed dressed rate $g_{\mathrm{eff}}/(2\pi) = 0.5\,\mathrm{MHz}$	5.00×10^{5} nat/s (assumed mapping)	159.2 ns	Assumed $\eta = 10$ bound: **4.61 μs**. Reported coupling: up to $g/(2\pi) = 329\,\mathrm{kHz}$; a matched compression-time measurement is not used in this comparison.
Trapped-ion fluorescence readout, $^{40}Ca^{+}$ [141]	Assumed collected-photon rate $\Gamma_{\mathrm{det}} = 8.0\times10^{4}\,\mathrm{s}^{-1}$	8.00×10^{4} nat/s (assumed mapping)	0.995 μs	Assumed $\eta = 10$ bound: **28.8 μs**. Reported adaptive readout: mean **145 μs** at 99.99% fidelity.

Remark (Numerical evaluation of the illustrative superconducting cQED baseline). Evaluating Eqs. (8.2)–(8.5) with Planck's constant $h = 4.135667696\times10^{-15}\,\mathrm{eV\cdot s}$ and $j_0 = 1/(4\pi) \approx 0.07957747$ for the illustrative superconducting circuit-QED baseline parameters ($\lvert\chi_{\mathrm{tot}}\rvert/(2\pi) = 10.0\,\mathrm{MHz}$ and mean intracavity photon number $\bar{n} = 9.0$) yields a pure entanglement interaction energy standard deviation $\Delta E_{\mathrm{int}} = h(\lvert\chi_{\mathrm{tot}}\rvert/2\pi)\sqrt{\bar{n}} = 0.1241\,\mu\mathrm{eV}$, a maximum channel capacity $C_{\max} = \Delta E_{\mathrm{int}}/h = 3.000\times10^{7}$ nat/s, a single-step resolution time scale $\Delta t_{\min}(j) = j/C_{\max} = 2.653\,\mathrm{ns}$, and a $\eta = 10$ continuous compression benchmark $\tau_{\mathrm{Nyq}}(\eta = 10) = \ln(10)/C_{\max} = 76.75\,\mathrm{ns}$. Across an illustrative parameter sweep chosen here ($\lvert\chi_{\mathrm{tot}}\rvert/(2\pi) \in [1, 30]\,\mathrm{MHz}$, $\bar{n} \in [1, 50]$, and $\eta \in [2, 50]$), both $\Delta t_{\min}(j)$ and τ_{Nyq} scale inversely with $\lvert\chi_{\mathrm{tot}}\rvert\sqrt{\bar{n}}$.

8.2 Prediction II: conditional phenomenological signatures of exploratory hypotheses H3–H4

Whereas Prediction I relies solely on the baseline Markovian evolution of Axiom III, Prediction II explores the empirical consequences if microscopic measurement channels additionally obey the exploratory non-Markovian subordination and discrete-dissipation hypotheses Assumptions H3 and H4 (Section 3.4). Under these hypotheses, discrete information exchange in elementary steps of $j = 1/(4\pi)$ nat produces three testable signatures beyond the standard Markovian stochastic master equation:

1. **Exploratory short-pulse commitment threshold (truncation knee) under H3.** In the baseline Markovian master equation (5.33), ensemble visibility decays linearly as $1 - V_{\mathrm{SME}}(\tau_{\mathrm{pulse}}) \approx \gamma_{\mathrm{info}}\tau_{\mathrm{pulse}}$ for arbitrarily short pulses $\tau_{\mathrm{pulse}} > 0$ (once cavity ring-up transients of duration $\tau_{\mathrm{cav}} \sim \kappa_{\mathrm{cav}}^{-1}$ have settled [20]). Under the strong-threshold subordination hypothesis of Assumption H3, however, let $P_{\mathrm{irr}}(\tau_{\mathrm{pulse}})$ denote the probability of an irreversible macroscopic record commitment after a rectangular interaction pulse of duration τ_{pulse} in a steady-state pre-populated probe mode. Because a pulse shorter than $\Delta t_{\min}(j) = \hbar/(2\Delta E_{\mathrm{int}})$ transfers cumulative mutual information $\Delta I_{\mathrm{ex}} < j$, Assumption H3 predicts that irreversible record commitment is suppressed below $\tau_{\mathrm{knee}} = \Delta t_{\min}(j)$, producing a flat commitment onset:

$$\begin{aligned} \left(\frac{dP_{\mathrm{irr}}}{d\tau_{\mathrm{pulse}}}\right)_{\tau_{\mathrm{pulse}}<\Delta t_{\min}(j)} &= 0, \\ \tau_{\mathrm{knee}} = \Delta t_{\min}(j) = \frac{\hbar}{2\Delta E_{\mathrm{int}}} &= \frac{1}{4\pi C_{\max}}. \end{aligned} \tag{8.6}$$

   Experimentally, whereas a finite-bandwidth cavity transient or quadratic Zeno onset in a pre-populated resonator exhibits a smooth $O(\tau_{\mathrm{pulse}}^2)$ rise for $0 < \tau_{\mathrm{pulse}} < \tau_{\mathrm{knee}}$ with characteristic time set by $\kappa_{\mathrm{cav}}^{-1}$, the hard-gate regime of Assumption H3 predicts a vanishing slope $dP_{\mathrm{irr}}/d\tau_{\mathrm{pulse}} = 0$ up to a knee $\tau_{\mathrm{knee}} \propto (|\chi_{\mathrm{tot}}|\sqrt{\bar{n}})^{-1}$ that shifts inversely with probe amplitude $\sqrt{\bar{n}}$ at fixed κ_{cav}.

2. **Symplectic-area staircase under unmonitored weak measurement backaction: stroboscopic tomography under H4.** When a ground-state harmonic oscillator is subjected to an unmonitored weak measurement backaction or phase-diffusion channel (or weakly coupled finite-temperature environment) that converts leaked mutual-information steps $\Delta I_{\mathrm{ex}} = kj$ into covariance differential entropy $\Delta S_{\mathrm{state}} = \Delta I_{\mathrm{ex}} = kj$ under Assumption H4, Theorem 3, Eq. (5.13), implies that the symplectic-area invariant $D_\Sigma = \sqrt{\Delta_q^2\Delta_p^2 - \sigma_{qp}^2}$, reconstructed by second-moment tomography, assumes discrete lattice values:

$$\begin{aligned} D_{\Sigma,k} &= \frac{\hbar}{2}\exp\left(\frac{k}{4\pi}\right) \\ \Longrightarrow \qquad \Lambda_k \equiv \ln\left(\frac{2D_{\Sigma,k}}{\hbar}\right) &= \frac{k}{4\pi} \approx 0.07958k, \qquad k = 0, 1, 2, \ldots. \end{aligned} \tag{8.7}$$

3. **Compression residence peaks and a heuristic quantum-efficiency benchmark in single homodyne trajectories under H3.** Under the weak-barrier modulation $\gamma_{\mathrm{eff}}(I_{\mathrm{ex}}) = \gamma_0[1 - \alpha_b\cos(2\pi I_{\mathrm{ex}}/j)]$ $(0 < \alpha_b < 1)$ of Assumption H3, continuous position monitoring of a Gaussian state produces histogram residence peaks in the logarithmic compression ratio $\Xi(t) \equiv \ln[\Delta_q(0)/\Delta_q(t)] = \Delta I_{\mathrm{ex}}(t)$ at the lattice values $\Xi_k = kj_0 = k/(4\pi) \approx 0.07958k$, with adjacent peak-to-trough contrast $(1 + \alpha_b)/(1 - \alpha_b)$. As a first-order heuristic resolution benchmark (prior to full Poisson/binomial photodetection filtering analysis), requiring the fractional uncollected-photon loss $1 - \eta_{\mathrm{det}}$ per unit nat and the mean thermal bath occupation $n_{\mathrm{th}} = (e^{\hbar\omega_0/k_BT} - 1)^{-1}$ not to exceed the half-step $j_0/2 = 1/(8\pi)$ gives the reference experimental window:

$$\begin{aligned} &\eta_{\mathrm{det}} > \eta_{\mathrm{det}}^{\mathrm{crit}} \equiv 1 - \frac{1}{8\pi} \approx 96.02\%, \\ &n_{\mathrm{th}} < n_{\mathrm{th}}^{\max} \equiv \frac{1}{8\pi} \approx 0.0398 \qquad (T < 88.2\,\mathrm{mK} \text{ at } 6\,\mathrm{GHz};\ \text{operated at } T \lesssim 15\,\mathrm{mK}). \end{aligned} \tag{8.8}$$

If high-statistics trajectory histograms within the regime of Eq. (8.8) remain smooth down to a specified modulation sensitivity $\alpha_b^{\min}$, the exploratory hypotheses H3/H4 are upper-bounded by $\alpha_b < \alpha_b^{\min}$, while the baseline Markovian structure of Axioms I–III remains unaffected.

Remark (Physical intuition and experimental requirements for testing H3 and H4). Because the elementary step $j = 1/(4\pi) \approx 0.0796\,\text{nat}$ is small, conventional readout chains with total collection efficiencies of 50%–80% lose 0.20–0.50 nat per extracted nat into unmonitored channels, washing out fine step structure of width $j/2 = 1/(8\pi) \approx 0.0398\,\text{nat}$. Testing the exploratory predictions of H3 and H4 requires near-quantum-limited amplification (such as traveling-wave parametric amplifiers or squeezed-state readout) to approach $\eta_{\text{det}} \gtrsim 96\%$, together with standard dilution-refrigerator temperatures ($T \lesssim 15\,\text{mK}$, well below the 88.2 mK thermal threshold for $n_{\text{th}} < 1/(8\pi)$ at $\omega_0/(2\pi) = 6\,\text{GHz}$).

8.3 Overview of the mathematical structure of the eleven QICM theorems

Table 6 summarizes the physical domains, core mathematical mechanisms, governing equations, and principal conclusions across Propositions 3.1–3.2 and Theorems 1–11.

Table 6: Mathematical structure and physical conclusions of Objective Phase-Space Quantum Information-Capacity Mechanics (QICM), synthesizing the core mechanisms, equations, and physical conclusions across Theorems 1–11.

Physical domain and central problem	Core QICM mechanism	Corresponding results and equations	Physical conclusion
Axiomatic structure and origin of quantum equilibrium	Chentsov–Hall–Reginatto uniqueness, quantum Cramér–Rao calibration, and coarse-grained H-theorem	Propositions 3.1–3.2, semantic Postulate S; Eqs. (3.12)–(3.15)	Selects the Fisher form under pointwise subsystem consistency; calibrates the postulated capacity $C_{\max} = \Delta E_{\text{int}}/h$; establishes conditional coarse-grained relaxation alongside envariance.
Phase-space geometry and operator equivalence	Kähler compatibility of Ω, Sasaki–Fisher–Rao metric G_{FR}, and J on $\mathcal{T}^{\text{red}}\mathcal{M}_{\text{ontic}}$	Theorems 1–2; Eqs. (4.1)–(4.10)	Establishes equivalence with the projective Hilbert-space Hermitian inner product, self-adjoint operator algebra, Poisson–commutator isomorphism, and premeasurement mapping.
Nonrelativistic kinematics and bound-state spectra	Cramér–Rao inequality, Fisher variational quantum potential, line-bundle circulation, and the codimension-1 nodal $\mathbb{Z}_2$ jump	Theorems 3–6; Eqs. (5.1)–(5.27)	Establishes uncertainty relations, the Gaussian-reference covariance-entropy bound, the resolution scale $\Delta t_{\min}(j)$, closed-limit Schrödinger and semiclassical equivalence, and bound-state oscillator, orbital, and hydrogenic spectra.

Physical domain and central problem	Core QICM mechanism	Corresponding results and equations	Physical conclusion
Open-measurement collapse and Born statistics	Unbiased Itô information updating constrained by $C_{\max,\alpha}$, Doob martingale convergence, and bare-coupling-rate hierarchy	Theorem 7; Eqs. (5.28a)–(5.34)	Connects closed flow and channel-driven collapse (globally on finite QND branches and up to resolution $\Delta_{\det} > 0$); establishes Born-weight consistency, the linear Lindblad equation, and the Shannon mutual-information identity.
Relativistic free fields and normal-mode excitations	Madelung–Klein–Gordon action and functional Schrödinger probability functional $P[\varphi, t]$ on field configuration space	Theorem 8; Eqs. (7.1)–(7.2)	Formulates covariant scalar equations and positive-definite free scalar and electromagnetic field functionals; relates standing-wave Fock number states to Hermite–Gaussian nodal surfaces.
Half-integer spin, fine structure, and statistical sectors	Half-integer holonomy on $\mathbb{R}^3 \times SO(3)$, Hopf coherent-section Kähler–Fisher trace, quotient manifold M_N with an explicit spin–statistics compatibility assumption, and imported Foldy–Wouthuysen terms	Theorem 9 and Corollary 9.1; Eqs. (7.3)–(7.5)	Obtains $\langle \boldsymbol{S}^2 \rangle = s(s+1)\hbar^2$ on coherent sections, 2π spinor sign reversal, local two-fermion radial Fisher repulsion, and a phase-space evaluation of $O(\alpha_{\mathrm{fs}}^2)$ hydrogen fine structure.
Ultraviolet singularities and the Compton cutoff	Phenomenological DBI-type ultraviolet effective functional at $\widetilde{\lambda}_C = \hbar/(mc)$ with amplitude-gradient ratio $I_{\mathrm{reg}}(\rho)$	Theorem 10; Eqs. (7.6)–(7.9)	Recovers quadratic Fisher kinetic energy at low energy and the free Klein–Gordon mass shell; bounds vortex-core speeds subluminally ($\lvert \boldsymbol{v}_{\mathrm{reg}} \rvert < c$) and calibrates nodal osmotic saturation to $2mc^2$ ($\xi_c = 1/8$).
Many-body no-signaling and boundary capacity bounds	Second-order Itô Fisher-drift identity, cancellation of diagonal-path damping, CPTP partial-trace identity, and elementary boundary-bond capacities	Theorem 11; Eqs. (7.10)–(7.13)	Establishes expected channel-backaction heating power, ensemble-level absence of superluminal signaling under general local CPTP operations, and a monitored boundary-channel capacity bound.

8.4 Scope and open frontiers

The established scope of Objective Phase-Space Quantum Information-Capacity Mechanics (QICM) and six open directions for further investigation are outlined below.

1. **Variational functionals for relativistic four-component chiral spinor fields, the QED Lamb shift, and non-Abelian Yang–Mills gauge fields.** Theorems 6, 8, and 9 and Corollary 9.1 establish the complete nonrelativistic hydrogen spectrum and $O(\alpha_{\rm fs}^2)$ relativistic fine-structure splitting, reconstruction of free scalar and transverse electromagnetic fields, and half-integer spin on the $SO(3)$ frame bundle. The consequence of Axiom I in Eq. (3.3) includes minimal coupling to a $U(1)$ electromagnetic gauge field. The full current of a relativistic four-component Dirac bispinor, the Lamb shift, namely the $2S_{1/2} - 2P_{1/2}$ splitting measured in hydrogen [142] and explained predominantly by the electron self-energy with an additional vacuum-polarization contribution [122, 143], parity-violating left-handed Weyl fermions, $SU(3)_C \times SU(2)_L \times U(1)_Y$ non-Abelian gauge fields, and constructive non-Gaussian measures and the scattering S-matrix of interacting quantum field theory in $3+1$ dimensions remain central subjects for further development.

2. **Dynamical gravitational coupling in curved spacetime and unification with the Bekenstein–Hawking holographic entropy bound.** Axiom II fixes the flat-phase-space dimensionless geometric constant $j_0 = 1/(4\pi)$, and Theorem 10 provides regularization at the reduced Compton wavelength $\widetilde{\lambda}_C = \hbar/(mc)$ for massive matter fields. A rigorous quantum-gravity extension is required to make the spacetime metric $g_{\mu\nu}(x)$ itself a dynamical field constrained by Planck-area information capacity, and to derive from horizon-channel capacity the Einstein equation $G_{\mu\nu} = (8\pi G/c^4)T_{\mu\nu}$, the Bekenstein–Hawking entropy bound, and the gravitational thermodynamic coefficient $1/4$, corresponding to $\pi j_0 = 1/4$ [144, 145].

3. **Global strong solutions of infinite-dimensional stochastic PDEs, relativistic hypersurface collapse, and vortex reconnection.** Theorem 7's preliminary lemma and explicit Bayesian solution establish global well-posedness for finitely many QND branches and stopping-time regularity up to any finite resolution $\Delta_{\rm det} > 0$ for continuous observables. For the general coupled infinite-dimensional stochastic PDEs (3.8a)–(3.8b) with continuous spectra and nodal singularities, existence and uniqueness of global strong solutions in weighted Sobolev spaces [18, 115], as well as 3D codimension-2 vortex reconnection rules, remain open problems in stochastic functional analysis. Likewise, while Theorem 8 formulates covariant closed field equations and Theorem 11 establishes ensemble no-signaling in a fixed Lorentz foliation, constructing a fully Lorentz-covariant single-trajectory stochastic collapse law across arbitrary spacelike hypersurfaces—analogous to the relativistic flash and field-reduction constructions of Tumulka [146] and Bedingham [147]—remains a key open problem.

4. **General phase-backaction unravelings, non-Markovian colored-noise channels, and two-dimensional anyon statistics.** Because Eq. (3.8a) with $d\Phi_{\rm back} = 0$ coincides with standard diffusive configuration filtering for any potential $V(q)$, further dynamical extensions include general heterodyne/phase-backaction unravelings ($d\Phi_{\rm back} \neq 0$) and microscopic apparatus channel-selection rules; non-Markovian colored-noise memory-kernel corrections for finite-bandwidth detectors; and extension of line-bundle holonomy to fractional-statistics anyons on the confined two-dimensional quotient manifold $M_N^{(2D)}$, whose fundamental group is the Artin braid group, $\pi_1 = B_N$ [148, 149].

5. **Nonperturbative many-body quantum-equilibrium relaxation rates and microscopic calibration of canonical thermal statistical mechanics.** Section 3.5 separates the envariance argument from a conditional coarse-grained relative-entropy H-theorem, Eq. (3.14).

The latter requires the stated diffusion closure, connected support, and an entropy-coercivity estimate for asymptotic relaxation; these properties are not consequences of common deterministic advection alone. Quantifying the quantum-equilibrium relaxation time in specific many-body chaotic Hamiltonian systems and establishing its microscopic correspondence with the eigenstate thermalization hypothesis (ETH) [150] remain open problems connecting quantum information ontology and nonequilibrium statistical mechanics.

6. **Logical separation and experimental testing of Predictions I and II.** Prediction I (Section 8.1, Eqs. (8.1)–(8.5)) tests the capacity bound $C_{\max} = \Delta E_{\mathrm{int}}/h$ within the baseline Markovian continuous-measurement framework, separating continuous Gaussian pointer compression ($\Delta I_{\mathrm{req}} = \ln \eta$) from discrete-branch Shannon discrimination ($\Delta I_{\mathrm{Sh}} \leq \ln 2$). Prediction II (Section 8.2, Eqs. (8.6)–(8.8)) explicitly isolates the additional exploratory assumptions H3 and H4, testing whether microscopic information transfer remains smooth and Markovian at all scales or exhibits non-Markovian short-pulse subordination knees at $\tau_{\mathrm{knee}} = \hbar/(2\Delta E_{\mathrm{int}})$ and discrete $k/(4\pi)$ nat staircases.

A Logical hierarchy and dependence of the theoretical claims

The mathematical framework of QICM has four logically distinct levels with traceable dependencies.

1. **Three axioms, uniqueness results, and probability semantics.** Axiom I sets out the foundational commitment that a microscopic system has no classical pointlike "true coordinate value" at any $t \in \mathbb{R}$ and specifies that—without assumptions on material composition or internal structure—its complete objective physical state relative to any other physical system is uniquely determined on M by the real statistical field pair $(\rho(\cdot,t), [\Phi(\cdot,t)]) \in \mathcal{M}_{\mathrm{ontic}}$, together with the canonical symplectic form Ω, quantized flat-$U(1)$ holonomy, and first-order nodal $\mathbb{Z}_2$ half-period jumps, Eqs. (3.1)–(3.2), providing an axiomatic topological closure of Wallstrom's objection, with electromagnetic minimal coupling in Eq. (3.3). Axiom II defines the geometric ratio $j_0 = 1/(4\pi)$, postulates the reference information-resolution step $j = 1/(4\pi)$ nat, and parameterizes $s_{\min} = hj_0 = \hbar/2$ and $\hbar = 2hj_0 = h/(2\pi)$, Eqs. (3.4)–(3.5). Axiom III supplies the Fisher kinetic-energy functional, the postulated Nyquist–Planck capacity bound $C_{\max} = \Delta E_{\mathrm{int}}/h$, and unified Itô evolution, Eqs. (3.6)–(3.11). Proposition 3.1 proves Fisher-form uniqueness under pointwise subsystem composition and calibrates $C_{\max}$ against the Braunstein–Caves speed limit, Eqs. (3.12)–(3.13). Probability postulate S, the envariance argument, and the conditional diffusion-and-coercivity route to quantum equilibrium are set out in Section 3.5, Eq. (3.14); Proposition 3.2 states open-measurement circulation and nodal-index conservation, Eq. (3.15).

2. **Kähler geometry, operator equivalence, and nonrelativistic results.** On the reduced horizontal tangent space $\mathcal{T}^{\mathrm{red}}\mathcal{M}_{\mathrm{ontic}}$ equipped with the Sasaki–Fisher–Rao cotangent metric G_{FR} and canonical symplectic form Ω, Axioms I–II and H1 yield Theorem 1, the Kähler equivalence of $(G_{\mathrm{FR}}, \Omega, J)$ with the projective Hilbert-space Hermitian product, Eqs. (4.1)–(4.7), and Theorem 2, self-adjoint generator representations, the Poisson–commutator isomorphism, and unitary premeasurement of nonconfiguration observables, Eqs. (4.8)–(4.10).

   Together with Axiom III, regularity H1, and measurement condition H2, these results support Theorems 3–7: momentum-variance decomposition, Robertson–Schrödinger uncertainty, and Gaussian-reference covariance-entropy bounds (Theorem 3, Eqs. (5.1)–(5.14)); the postulated channel-resolution time scale for a reference information increment (Theorem 4, Eqs. (5.15)–(5.18)); closed-limit Schrödinger equivalence, semiclassical Hamilton–Jacobi reduction, and Gaussian Ermakov–Pinney dynamics (Theorem 5, Eqs. (5.19)–(5.24)); bound-state oscillator,

orbital, and hydrogenic Rydberg spectra with Bogomol'nyi ground-state uniqueness (Theorem 6, Eqs. (5.25)–(5.27)); and variance decay, Born-weight consistency under unbiased martingale updates, linear Lindblad evolution, and finite-branch Shannon mutual-information identities (Theorem 7, Eqs. (5.28a)–(5.34)). These results provide the real-field accounts of double-slit duality, EPR–CHSH violations, and Zeno bounds in Section 6, Eqs. (6.1)–(6.5), and the Markovian capacity bounds and matched-parameter dispersive cQED test of Prediction I in Section 8.1, Eqs. (8.1)–(8.5).

3. **Relativistic free fields, spin and statistics, fine structure, ultraviolet regularization, and many-body no-signaling.** Applying Axioms I–III and Theorems 1–2 to Minkowski spacetime, field configuration space, the $SO(3)$ frame bundle, and the identical-particle quotient M_N yields Theorems 8–11: covariant Madelung–Klein–Gordon equations and positive-definite free scalar and transverse electromagnetic field functionals (Theorem 8, Eqs. (7.1)–(7.2)); half-integer spin holonomy, the Hopf coherent-section identity $\langle \boldsymbol{S}^2 \rangle = s(s+1)\hbar^2$, local two-fermion radial Fisher repulsion, and a phase-space evaluation of imported Foldy–Wouthuysen hydrogen fine-structure corrections (Theorem 9 and Corollary 9.1, Eqs. (7.3)–(7.5)); a phenomenological Compton-scale DBI ultraviolet regularization with free Klein–Gordon mass-shell consistency, subluminal vortex-core drift, and calibrated $2mc^2$ nodal saturation at $\xi_c = 1/8$ (Theorem 10, Eqs. (7.6)–(7.9)); and expected second-order Fisher-information Itô drift, general local CPTP ensemble no-signaling, and the monitored boundary-channel capacity bound (Theorem 11, Eqs. (7.10)–(7.13)). The identification of spin and exchange holonomies in Theorem 9 additionally requires its stated compatibility assumption; it is not derived from Axioms I–III alone.

4. **Exploratory predictions requiring Assumptions H3 and H4.** In Section 8.2, Prediction II [Eqs. (8.6)–(8.8)] addresses the non-Markovian short-pulse commitment threshold $\tau_{\mathrm{knee}} = \hbar/(2\Delta E_{\mathrm{int}})$, weakly dissipative symplectic-area quantization, residence peaks in high-efficiency homodyne trajectories, and the heuristic resolution benchmark $\eta_{\mathrm{det}}^{\mathrm{crit}} \approx 96.02\%$. Its discrete information-clock subordination assumption H3 and equal information–state-entropy conversion assumption H4 are independent exploratory hypotheses, logically separate from the baseline structure in levels 1–3.

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